CoCo 2026 Participant: CeTA 3.9

Christina Kirk and René Thiemann

University of Innsbruck, Austria

The tool CeTA [1] is a certifier for, among other properties, (non-)confluence of term rewrite systems (TRSs) with and without conditions, (non-)commutation of TRSs and (in)feasibility of conditions w.r.t. some conditional TRSs. Soundness is proven as part of the formal proof library IsaFoR, the Isabelle Formalization of Rewriting. For a complete reference of supported techniques we refer to the certification problem format (CPF) and the IsaFoR/CeTA website:

https://isafor-ceta.uibk.ac.at/

We here describe only the most relevant change of CeTA 3.9 in comparison to the version of the previous year: It is the full support of Okui's confluence criterion [2]. Previously, only the soundness statement was proven [3], without an algorithm to compute the set of simultaneous critical pairs (SCPs) for a TRS. In CeTA 3.9 such an algorithm is now implemented, verified, and used to to develop a certificate checker for Okui's criterion [4]. The checker expects certificates as follows, where rewrite sequences $\to^*$ can be represented succinctly via multisteps $\multimap^*$.

- For every non-trivial SCP $s \leftarrow\!\circ\!- \cdot \to t$ (i.e., $s \neq t$), a joining sequence needs to be provided, by writing down the intermediate terms $u_1, \ldots, u_n$ in the certificate.

$$s \multimap u_1 \multimap u_2 \multimap \ldots \multimap u_n \leftarrow\!\circ\!- t$$

CeTA also supports commutation proofs via a generalization of Okui's criterion [4]. Given two left-linear TRSs $\mathcal{R}$ and $\mathcal{S}$, in order to prove strong commutation of $\multimap_\mathcal{R}$ and $\multimap_\mathcal{S}$, we require certificates that have the same design as in the confluence setting.

- For every non-trivial SCP $s \,{}_\mathcal{R}\!\leftarrow\!\circ\!- \cdot \to_\mathcal{S} t$, a joining sequence needs to be provided.

$$s \multimap_\mathcal{S} u_1 \multimap_\mathcal{S} u_2 \multimap_\mathcal{S} \ldots \multimap_\mathcal{S} u_n \,{}_\mathcal{R}\!\leftarrow\!\circ\!- t$$

References

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[2] Satoshi Okui. Simultaneous critical pairs and Church-Rosser property. In Tobias Nipkow, editor, *Rewriting Techniques and Applications, 9th International Conference, RTA-98, Tsukuba, Japan, March 30 - April 1, 1998, Proceedings*, volume 1379 of *Lecture Notes in Computer Science*, pages 2–16. Springer, 1998. URL: https://doi.org/10.1007/BFb0052357, doi:10.1007/BFB0052357.

[3] Christina Kirk and Aart Middeldorp. Formalizing simultaneous critical pairs for confluence of left-linear rewrite systems. In Kathrin Stark, Amin Timany, Sandrine Blazy, and Nicolas Tabareau, editors, *Proceedings of the 14th ACM SIGPLAN International Conference on Certified Programs and Proofs, CPP 2025, Denver, CO, USA, January 20-21, 2025*, pages 156–170. ACM, 2025. doi:10.1145/3703595.3705881.

[4] René Thiemann. Verifying and Generalizing Simultaneous Critical Pairs. *Proceedings of the 15th International Workshop on Confluence, July 24, 2026.* This volume.