

CRaris (Version 1.2)

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CRaris, a **CR** checker for LCTRSs in **ari** style,¹ is a tool to prove confluence of *logically constrained term rewrite systems* (LCTRSs, for short) [4] written in the ARI format [1].² The tool is based on Crisys2³—Constrained rewriting induction system (version 2)—and receives LCTRSs written in the ARI format only to prove confluence, while Crisys2 has many functions to, e.g., solve *all-path reachability problems* [2]. To prove confluence of LCTRSs, the tool uses the following criteria:

- weak orthogonality [4], and
- termination and joinability of critical pairs [7].

To prove termination, the tool uses the DP framework for LCTRSs [3] without any interpretation method, together with a criterion for LCTRSs with bitvector arithmetics [5].

The *critical pairs* of two constrained rewrite rules $\rho_1 : \ell_1 \rightarrow r_1 [\varphi_1]$ and $\rho_2 : \ell_2 \rightarrow r_2 [\varphi_2]$ with distinct variables (i.e., $\text{Var}(\ell_1, r_1, \varphi_1) \cap \text{Var}(\ell_2, r_2, \varphi_2) = \emptyset$) are all tuples $\langle s, t, \phi \rangle$ such that a non-variable subterm $\ell_1|_p$ of ℓ_1 at a position p is unifiable with ℓ_2 , “ $p \neq \varepsilon$, $\rho_1 \neq \rho_2$ up to variable renaming, or $\text{Var}(r_1) \subseteq \text{Var}(\ell_1)$ ”, the most general unifier γ of $\ell_1|_p$ and ℓ_2 respects variables of both ρ_1 and ρ_2 , i.e., $\gamma(x)$ is either a value or a variable for all variables x in $\text{Var}(\varphi_1, \varphi_2) \cup (\text{Var}(r_1, r_2) \setminus \text{Var}(\ell_1, \ell_2))$, $(\varphi_1 \wedge \varphi_2)\gamma$ is satisfiable, $s = r_1\gamma$, $t = (\ell_1[r_2]_p)\gamma$, and $\phi = (\varphi_1 \wedge \varphi_2)\gamma$. The set of critical pairs of an LCTRS \mathcal{R} is denoted by $CP(\mathcal{R})$, which includes all critical pairs of two rules in $\mathcal{R} \cup \mathcal{R}_{calc}$. A critical pair $\langle s, t, \phi \rangle$ is called *trivial* if $s[\phi] \sim t[\phi]$. An LCTRS \mathcal{R} is called *weakly orthogonal* if \mathcal{R} is left-linear and all critical pairs of \mathcal{R} are trivial.

Theorem 1 ([4]). *A weakly orthogonal LCTRS is confluent.*

A critical pair $\langle s, t, \phi \rangle$ is called *joinable* if $(\langle s, t \rangle [\phi]) \rightarrow_{\mathcal{R}}^* (\langle s', t' \rangle [\phi'])$ and $s'[\phi'] \sim t'[\phi']$.

Theorem 2 ([7]). *A terminating LCTRS is confluent if all its critical pairs are joinable.*

This version uses C03 [6]⁴—a **CO**nverter for proving **CO**nfluence of **CO**nditional TRSs—to prove confluence of LCTRSs that can be considered *many-sorted TRSs* (MS-TRSs, for short): An LCTRS \mathcal{R} is an MS-TRS if every rule $\ell \rightarrow r \Leftarrow \varphi$ satisfies all of the following:

- both ℓ and r are calculation-free,
- $\text{Var}(\ell) \supseteq \text{Var}(r)$, and
- φ is true.

For example, the LCTRS 1526.ari in the ARI database can be considered an MS-TRS. Since we consider well-typed LCTRSs only, the MS-TRSs obtained from LCTRSs are well-typed. Since the techniques used in C03 do not rely on sorts, we transform an LCTRS considered an MS-TRS into a TRS by dropping sorts and guards.

¹<http://www.lctrs.jp/tools/craris/>

²<https://project-coco.uibk.ac.at/ARI/lctrs.php>

³<https://www.lctrs.jp/tools/crisys2/>

⁴<https://www.lctrs.jp/tools/co3/>

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