CoCo 2025 Participant: CeTA 3.6

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The tool CeTA [6] is a certifier for, among other properties, (non-)confluence of term rewrite systems with and without conditions. Its soundness is proven as part of the formal proof library IsaFoR, the Isabelle Formalization of Rewriting. Below, we present the relevant changes from last year's version (3.1) to this year's version (3.6). For a complete reference of supported techniques we refer to the certification problem format (CPF) and the IsaFoR/CeTA website:

http://cl-informatik.uibk.ac.at/isafor/

CeTA 3.6 has the new feature that it is capable of checking *feasibility* proofs, so that now in the INF category of CoCo both YES-answers and NO-answers can be certified. Essentially, a certificate of a feasibility proof consists of the substitution that proves feasibility in combination with details on the rewrite sequence, consisting of conditional rewrite steps.

In CeTA 3.6, also a new class of term orderings have been added. These orderings can be used in non-joinability proofs as discrimination pairs, or in infeasibility proofs as co-rewrite pairs. The new class of term orderings are Hofbauer and Waldmann's core matrix interpretations [2]. We generalized these orderings from the SRS version to a full TRS version [4]. Note that core matrix interpretations have slightly different requirements than the matrix interpretations of Endrullis et al. [1].

Regarding non-commutation of two TRSs $\mathcal{R}$ and $\mathcal{S}$, CeTA 3.6 has added a swap technique, so that the role of $\mathcal{R}$ and $\mathcal{S}$ can be swapped. The reason is that some non-commutation techniques are not symmetric. Previously, swapping was only supported within commutation proofs.

A further significant addition has been added to IsaFoR, namely in the form of Okui's confluence criterion [3, 5]. However, this part is not yet available in CeTA: it remains to develop and verify an algorithm to compute all simultaneous critical pairs of TRSs.

References

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