CO3 (Version 2.6)

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CO3, a **co**nverter for proving **co**nfluence of **co**nditional TRSs,[1] tries to prove confluence of conditional term rewrite systems (CTRSs, for short) by using a transformational approach (cf. [8]). The tool first transforms a given weakly-left-linear (WLL, for short) 3-DCTRS into an unconditional term rewrite system (TRS, for short) by using $\mathbb{U}_{conf}$ [3], a variant of the *unraveling* $\mathbb{U}$ [10], and then verifies confluence of the transformed TRS by using the following theorem: A 3-DCTRS $\mathcal{R}$ is confluent if $\mathcal{R}$ is WLL and $\mathbb{U}_{conf}(\mathcal{R})$ is confluent [2, 3]. The tool is very efficient because of very simple and lightweight functions to verify properties such as confluence and termination of TRSs.

Since version 2.0, a *narrowing-tree*-based approach [9, 4] to prove infeasibility of a condition w.r.t. a CTRS has been implemented [5]. The approach is applicable to *syntactically deterministic* CTRSs that are operationally terminating and *ultra-right-linear* w.r.t. the *optimized* unraveling. To prove infeasibility of a condition c, the tool first proves confluence, and then linearizes c if failed to prove confluence; then, the tool computes and simplifies a narrowing tree for c, and examines the emptiness of the narrowing tree. Since version 2.2, CO3 accepts both *join* and *semi-equational* CTRSs, and transforms them into equivalent DCTRSs to prove confluence or infeasibility [6].

The difference from the previous version [7] is a slight improvement of the subterm criterion. CO3 uses very lightweight criteria for proving termination, while using the DP framework [1]. The *subterm criterion* implemented in the previous version considers the first argument of marked symbols, while arbitrary arguments can be taken. This version uses the second argument in addition to the first one: The subterm criterion processor tries to prove finiteness of a given DP problem by means of the first argument, and if failed, then it tries it by means of the second argument. This slight improvements succeeds in proving termination of (C)TRSs and thus confluence of, e.g., `1009.ari`.

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