v0.11 (Kawano, Hirokawa, Shintani, JAIST)

confluence tool for left-linear TRSs

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2. rule removal by rule labeling & critical pair systems — (LMCS & IWC 2024)

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Theorem (critical pair system)

$\mathsf{CR}(\mathcal{R}) \Leftrightarrow \mathsf{CR}(\mathcal{S})$ if $\mathcal{R}$ is LL, $\mathcal{S} \subseteq \mathcal{R}$, $\mathsf{PCP}(\mathcal{R}) \subseteq \downarrow_{\mathcal{R}}$, $\mathsf{SN}(\mathcal{P}/\mathcal{R})$, and $\mathcal{R}\restriction_{\mathcal{S}} \subseteq \to_{\mathcal{S}}^{*}$

- $\mathcal{P} = \left\{ \begin{matrix} s \to t \\ s \to u \end{matrix} \;\middle|\; t \underset{\mathcal{R}}{\leftarrow\!\!\!\!\!\!+\!\!\!\!\!\!+} s \xrightarrow[\mathcal{R}]{\epsilon} u \text{ is parallel critical peak but not } t \xleftrightarrow[\mathcal{S}]{*} u \right\}$
- $\mathcal{R}\restriction_{\mathcal{S}} = \{\ell \to r \in \mathcal{R} \mid \mathcal{F}\mathsf{un}(\ell) \subseteq \mathcal{F}\mathsf{un}(\mathcal{S})\}$

Confluence Proof by Rule Removal

consider left-linear TRS $\mathcal{R}$:

$$\mathsf{s}(\mathsf{p}(x)) \xrightarrow{\mathbf{1}} \mathsf{p}(\mathsf{s}(x)) \qquad \mathsf{p}(\mathsf{s}(x)) \xrightarrow{\mathbf{2}} x \qquad \infty \xrightarrow{\mathbf{3}} \mathsf{s}(\infty)$$

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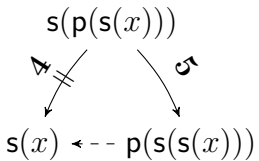

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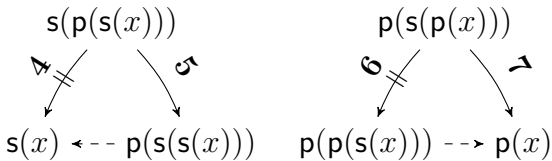

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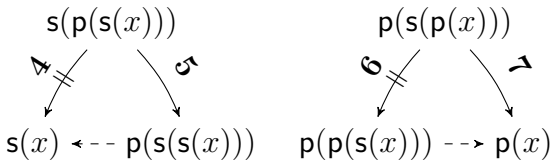

$\mathsf{SN}(\{\mathbf{4}, \mathbf{5}, \mathbf{6}, \mathbf{7}\}/\mathcal{R})$

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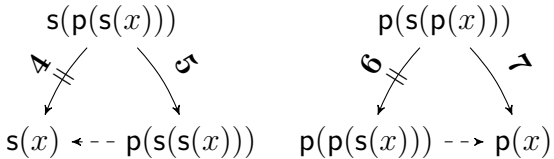

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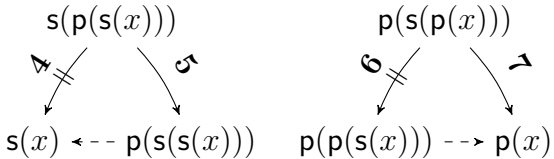

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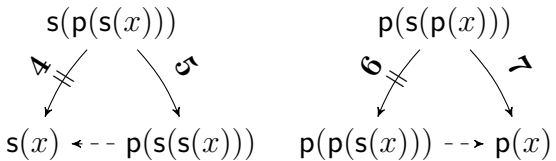

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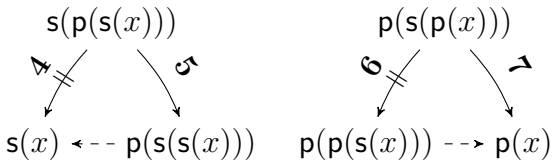

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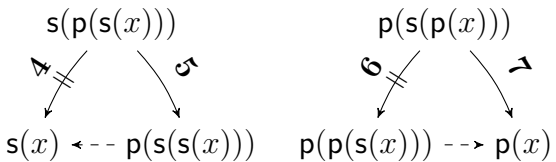

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3 $\mathsf{CR}(\varnothing)$ is trivial