Moca 0.3: A First-Order Theorem Prover for Horn Clauses

Yusuke Oi, Nao Hirokawa, and Teppei Saito

JAIST, Japan

Overview Moca is a fully automatic first-order theorem prover for Horn clauses. The tool, written in Haskell, is freely available from:

http://www.jaist.ac.jp/project/maxcomp/

The usage is: `moca.sh <file>`. Given a satisfiability problem in the TPTP CNF format [5], the tool outputs `Satisfiable` or `Unsatisfiable` if its satisfiability or unsatisfiability is proved, respectively, and `Maybe` otherwise. Given an infeasibility problem in the ARI format, the tool outputs `YES` if its infeasibility is proved, and `MAYBE` otherwise.

New feature This version of Moca can generate *certifiable* infeasibility proofs in the CPF 3 format. Using this feature, Moca forms a coalition with the certifier CeTA to give new certified proofs to a number of problems in the infeasibility category.

Techniques Moca implements *maximal ordered completion* [7] together with *approximation* techniques [4], the generalized *split-if* encoding [1, 4] (akin to unraveling [3, 2]), and inlining for conditional rewrite rules [6]. With a small example we illustrate how Moca uses them to solve problems. Consider the infeasibility problem of the conversion $x - x \leftrightarrow^* \mathsf{s}(x)$ for the TRS:

$$x - \mathsf{0} \to x \qquad \mathsf{0} - x \to \mathsf{0} \qquad \mathsf{s}(x) - \mathsf{s}(y) \to x - y$$

The problem can be regarded as the satisfiability problem of the Horn clauses:

$$x - \mathsf{0} \approx x \qquad \mathsf{0} - x \approx \mathsf{0} \qquad \mathsf{s}(x) - \mathsf{s}(y) \approx x - y \qquad x - x \not\approx \mathsf{s}(x)$$

By applying the split-if encoding the problem reduces to the word problem of deciding $\mathsf{T} \not\approx_{\mathcal{E}} \mathsf{F}$ for the equational system $\mathcal{E}$:

$$x - \mathsf{0} \approx x \qquad \mathsf{0} - x \approx \mathsf{0} \qquad \mathsf{s}(x) - \mathsf{s}(y) \approx x - y \qquad \mathsf{f}(\mathsf{s}(x), x) \approx \mathsf{F} \qquad \mathsf{f}(x - x, x) \approx \mathsf{T}$$

In order to solve it our tool attempts to construct a ground-complete presentation of $\mathcal{E}$ by using maximal ordered completion. However, the attempt is doomed to fail as the completion diverges. Moca overcomes the divergence by approximating the last equation to the more general equation $\mathsf{f}(x - x, y) \approx \mathsf{T}$. This results in the following equational system:

$$x - \mathsf{0} \approx x \qquad \mathsf{0} - x \approx \mathsf{0} \qquad \mathsf{s}(x) - \mathsf{s}(y) \approx x - y \qquad \mathsf{f}(\mathsf{s}(x), x) \approx \mathsf{F} \qquad \mathsf{f}(x - x, y) \approx \mathsf{T}$$

Now maximal ordered completion builds up the finite ground-complete presentation $\mathcal{R}$ of the approximated equational system:

$$\begin{array}{ccc} x - \mathsf{0} \to x & \mathsf{0} - x \to \mathsf{0} & \mathsf{s}(x) - \mathsf{s}(y) \to x - y \\ \mathsf{f}(\mathsf{0}, y) \to \mathsf{T} & \mathsf{f}(\mathsf{s}(x), x) \to \mathsf{F} & \mathsf{f}(x - x, y) \to \mathsf{T} \end{array}$$

Since $\mathsf{T}{\downarrow_{\mathcal{R}}} \neq \mathsf{F}{\downarrow_{\mathcal{R}}}$ holds, infeasibility of the conversion $x - x \leftrightarrow^* \mathsf{s}(x)$ is concluded.

References

[1] K. Claessen and N. Smallbone. Efficient Encodings of First-Order Horn Formulas in Equational Logic. In *Proc. 9th IJCAR*, LNCS 10900, pages 388–404, 2018.

[2] K. Gmeiner, N. Nishida and B. Gramlich. Proving Confluence of Conditional Term Rewriting Systems via Unravelings In *Proc. 2nd IWC*, pages 35–41, 2013.

[3] M. Marchiori. Unravelings and Ultra-Properties. In *Proc. 5th ALP*, LNCS 1139, pages 107–21, 1996.

[4] Y. Oi. Refutation by Completion and Approximations. Master's thesis, JAIST, 2019.

[5] G. Sutcliffe. The TPTP Problem Library and Associated Infrastructure: From CNF to TH0, TPTP v6.4.0. *Journal of Automated Reasoning*, 59(4):483–502, 2017.

[6] C. Sternagel and T. Sternagel. Certifying Confluence of Quasi-Decreasing Strongly Deterministic Conditional Term Rewrite Systems. In *Proc. 26th CADE*, LNCS 10395, pp. 413–431, 2017.

[7] S. Winkler and G. Moser. MædMax: A Maximal Ordered Completion Tool. In *Proc. 9th IJCAR*, LNCS 10900, pages 472–480, 2018.