CRaris: CR checker for LCTRSs in ARI Style

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CRaris, a **CR** checker for LCTRSs in **ARI s**tyle,[1] is a tool to prove confluence of *logically constrained term rewrite systems* (LCTRSs, for short) [4] written in ARI format [1].[2] The tool is based on Crisys2, **c**onstrained **r**ewriting **i**nduction **sys**tem (version **2**),[3] and receives LCTRSs written in ARI format only to prove confluence, while crisys2 has many functions to e.g., solve *all-path reachability problems* [2]. To prove confluence of LCTRSs, the tool uses the following criteria:

- weak orthogonality [4], and
- termination and joinability of critical pairs [6].

To prove termination, the tool uses the DP framework for LCTRSs [3] without any interpretation method, together with a criterion for LCTRSs with bitvector arithmetics [5].

The *critical pairs* of two constrained rewrite rules $\rho_1 : \ell_1 \to r_1\ [\varphi_1]$ and $\rho_2 : \ell_2 \to r_2\ [\varphi_2]$ with distinct variables (i.e., $\mathcal{V}ar(\ell_1, r_1, \varphi_1) \cap \mathcal{V}ar(\ell_2, r_2, \varphi_2) = \emptyset$) are all tuples $\langle s, t, \phi\rangle$ such that a non-variable subterm $\ell_1|_p$ of ℓ_1 at a position p is unifiable with ℓ_2, "$p \neq \varepsilon$, $\rho_1 \neq \rho_2$ up to variable renaming, or $\mathcal{V}ar(r_1) \subseteq \mathcal{V}ar(\ell_1)$", the most general unifier γ of $\ell_1|_p$ and ℓ_2 respects variables of both ρ_1 and ρ_2, i.e., $\gamma(x)$ is either a value or a variable for all variables x in $\mathcal{V}ar(\varphi_1, \varphi_2) \cup (\mathcal{V}ar(r_1, r_2) \setminus \mathcal{V}ar(\ell_1, \ell_2))$, $(\varphi_1 \wedge \varphi_2)\gamma$ is satisfiable, $s = r_1\gamma$, $t = (\ell_1[r_2]_p)\gamma$, and $\phi = (\varphi_1 \wedge \varphi_2)\gamma$. The set of critical pairs of an LCTRS $\mathcal{R}$ is denoted by $CP(\mathcal{R})$, which includes all critical pairs of two rules in $\mathcal{R} \cup \mathcal{R}_{calc}$. A critical pair $\langle s, t, \phi\rangle$ is called *trivial* if $s\ [\phi] \sim t\ [\phi]$. An LCTRS $\mathcal{R}$ is called *weakly orthogonal* if $\mathcal{R}$ is left-linear and all critical pairs of $\mathcal{R}$ are trivial.

Theorem 1 ([4]). *A weakly orthogonal LCTRS is confluent.*

A critical pair $\langle s, t, \phi\rangle$ is called *joinable* if $(\langle s, t\rangle\ [\phi]) \to^*_{\mathcal{R}} (\langle s', t'\rangle\ [\phi'])$ and $s'\ [\phi'] \sim t'\ [\phi']$.

Theorem 2 ([6]). *A terminating LCTRS is confluent if all its critical pairs are joinable.*

Instead of joinability of critical pairs, the tool uses a much simpler sufficient condition.

Proposition 3. *A critical pair $\langle s, s, \phi\rangle$ is trivial and thus joinable.*

We use "$s = t$" as a sufficient condition for triviality of $\langle s, s, \phi\rangle$.

References

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