NaTT in CoCo 2022

Akihisa Yamada @ AIST

NaTT in CoCo

- NaTT a **termination prover** for **plain** term rewriting
- In CoCo 2021:
  - participated **INF** category with an easy checker [Sternagel & Yamada '19]
  - ignores conditions
  - weak but fast
- In CoCo 2022:
  - Generalized reduction pairs for INF [IJCAR '22]
  - don't ignore conditions
  - not too weak, but slow

Orderings for non-reachability

reduction pair [*ver.* Y, CADE '21]:

- $\succcurlyeq$ and $\succ$ are closed under substitutions
- $\succcurlyeq$ is closed under contexts
- $\succcurlyeq$ is a quasi-order
- $\succ$ is well-founded (order)
- $\succcurlyeq \cdot \succ \subseteq \succ$ **and** $\succ \cdot \succcurlyeq \subseteq \succ$
- $\succ \subseteq \succcurlyeq$

Orderings for non-reachability

rewrite pair [Y, IJCAR '22]:

- $\succcurlyeq$ and $\succ$ are closed under substitutions
- $\succcurlyeq$ is closed under contexts
- $\succcurlyeq$ is a quasi-order
- $\succ$ **is irreflexive** (order)
- $\succcurlyeq \cdot \succ \subseteq \succ$ and $\succ \cdot \succcurlyeq \subseteq \succ$
- $\succ \subseteq \succcurlyeq$

Orderings for non-reachability

rewrite pair [Y, IJCAR '22]:

- $\succcurlyeq$ and $\succ$ are closed under substitutions
- $\succcurlyeq$ is closed under contexts
- $\succcurlyeq$ is a quasi-order
- $\succ$ **is irreflexive** (order)
- $\succcurlyeq \cdot \succ \subseteq \succ$ and $\succ \cdot \succcurlyeq \subseteq \succ$
- $\succ \subseteq \succcurlyeq$

Theorem:

$s \twoheadrightarrow t$ is $\mathcal{R}$-unsat **if** there's a rewrite pair $\langle \succcurlyeq, \succ \rangle$ s.t. $\mathcal{R} \subseteq \succcurlyeq$ and $s \prec t$

Orderings for non-reachability

co-rewrite pair [Y, IJCAR '22]:

- $\succcurlyeq$ and $\prec$ are closed under substitutions
- $\succcurlyeq$ is closed under contexts
- $\succcurlyeq$ is a quasi-order
- ($\prec$ is irreflexive)
- $\succcurlyeq \cap \prec = \emptyset$

Theorem:

$s \twoheadrightarrow t$ is $\mathcal{R}$-unsat **iff** there's a co-rewrite pair $\langle \succcurlyeq, \prec \rangle$ s.t. $\mathcal{R} \subseteq \succcurlyeq$ and $s \prec t$

Corollary:

$s \twoheadrightarrow t$ is $\mathcal{R}$-unsat **iff** $\mathcal{R}$ has a quasi-model $\langle \mathcal{A}, \geq \rangle$ s.t. $\mathcal{A} \vDash s \ngeq t$

Co-WPO

$s \sqsupseteq_{\text{WPO}} t$

1. $\mathcal{A} \vDash s > t$ or
2. $\mathcal{A} \vDash s \geq t$ and
   a. $\exists i \in \pi(f).\, s_i \sqsupseteq_{\text{WPO}} t$; or
   b. $\forall j \in \pi(g).\, s \sqsupset_{\text{WPO}} t_j$ and
      i. $f \succ g$ or
      ii. $f \succsim g$ and $\pi_f(s_1, \ldots, s_n) \sqsupseteq^{\text{lex}}_{\text{WPO}} \pi_g(t_1, \ldots, t_m)$;
   c. $s = t \in \mathcal{V}$

$t \sqsupset_{\overline{\text{WPO}}} s$

1. $\mathcal{A} \vDash t \nleq s$ or
2. $\mathcal{A} \vDash t \nless s$ and
   a. $\exists j \in \pi(g).\, t_j \sqsupseteq_{\overline{\text{WPO}}} s$; or
   b. $\forall i \in \pi(f).\, t \sqsupset_{\overline{\text{WPO}}} s_i$ and
      i. $g \npreceq f$ or
      ii. $g \nprec f$ and $\pi_g(t_1, \ldots, t_m) \sqsupset^{\text{lex}}_{\overline{\text{WPO}}} \pi_f(s_1, \ldots, s_n)$;

Theorem [Y, IJCAR '22]:

$\langle \sqsupseteq_{\text{WPO}}, \sqsubset_{\overline{\text{WPO}}} \rangle$ is a co-rewrite pair (under mild condition).