Toma 0.2: An Equational Theorem Prover

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Toma is an automatic theorem prover for first-order equational systems, freely available at:

https://www.jaist.ac.jp/project/maxcomp/

The typical usage is: `toma --inf <file>`, where `<file>` is an infeasibility problem in the CoCo format [5]. The tool outputs `YES` if infeasibility of the problem is shown, and `MAYBE` otherwise. It also accepts the TPTP CNF format [6].

Toma proves infeasibility as follows: By using the *split-if* encoding [2] a given infeasibility problem is transformed into a word problem of form $\mathcal{E} \vdash \mathsf{T} \not\approx \mathsf{F}$ whose validity entails infeasibility of the original problem. The word problem is solved by a new variant of maximal (ordered) completion [7, 3]:

1. Given an equational system $\mathcal{E}_1$, we construct a lexicographic path order $\succ_{\mathsf{lpo}}$ that maximizes reducibility of the ordered rewrite system $(\mathcal{E}_1, \succ_{\mathsf{lpo}})$ [7].
2. Using the order, we run ordered completion [1] on $\mathcal{E}_1$. Here we do not employ the deduce rule (critical pair generation). Such a run eventually ends with an inter-reduced version $(\mathcal{E}_2, \succ_{\mathsf{lpo}})$ of $(\mathcal{E}_1, \succ_{\mathsf{lpo}})$.
3. The tool checks ground-completeness of the ordered rewrite system $(\mathcal{E}_2, \succ_{\mathsf{lpo}})$ by Martin and Nipkow's method [4].
   (a) If $(\mathcal{E}_2, \succ_{\mathsf{lpo}})$ is ground-complete but T and F are not joinable, the tool outputs `YES` and terminates.
   (b) If T and F are joinable in $(\mathcal{E}_2, \succ_{\mathsf{lpo}})$, the tool outputs `MAYBE` and terminates.
   (c) Otherwise, there exists at least one equation that is valid in $\mathcal{E}_2$ but not ground-joinable in $(\mathcal{E}_2, \succ_{\mathsf{lpo}})$. Let $\mathcal{E}_3$ be a set of such equations. Setting $\mathcal{E}_1 := \mathcal{E}_2 \cup \mathcal{E}_3$, the tool goes back to the first step.

References

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