Moca 0.2: A First-Order Theorem Prover for Horn Clauses

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Moca is a fully automatic first-order theorem prover for Horn clauses. The tool, written in Haskell, is freely available from:

http://www.jaist.ac.jp/project/maxcomp/

The usage is: `moca.sh <file>`. Given a satisfiability problem in the TPTP CNF format [5], the tool outputs `Satisfiable` or `Unsatisfiable` if its satisfiability or unsatisfiability is proved, respectively, and `Maybe` otherwise. Given an infeasibility problem in the CoCo format [2], the tool outputs `YES` if its infeasibility is proved, and `MAYBE` otherwise.

Moca implements *maximal ordered completion* [6] and new *approximation* techniques. With a small example we illustrate how Moca uses them to solve problems. Consider the infeasibility problem of the conversion $x-x \leftrightarrow^* \mathsf{s}(x)$ for the TRS $\{x-\mathsf{0} \to x, \mathsf{0}-x \to \mathsf{0}, \mathsf{s}(x)-\mathsf{s}(y) \to x-y\}$. The problem can be regarded as the satisfiability problem of the Horn clauses:

$$x-\mathsf{0} \approx x \qquad \mathsf{0}-x \approx \mathsf{0} \qquad \mathsf{s}(x)-\mathsf{s}(y) \approx x-y \qquad x-x \not\approx \mathsf{s}(x)$$

By applying the *split-if* encoding [1] the problem reduces to the word problem of deciding $\mathsf{T} \not\approx_{\mathcal{E}} \mathsf{F}$ for the equational system $\mathcal{E}$:

$$x-\mathsf{0} \approx x \quad \mathsf{0}-x \approx \mathsf{0} \quad \mathsf{s}(x)-\mathsf{s}(y) \approx x-y \quad \mathsf{f}(\mathsf{s}(x),x) \approx \mathsf{F} \quad \mathsf{f}(x-x,x) \approx \mathsf{T}$$

In order to solve it our tool attempts to construct a ground-complete presentation of $\mathcal{E}$ by using maximal ordered completion. However, the attempt is doomed to fail as the completion diverges. Moca overcomes the divergence by approximating the last equation to the more general equation $\mathsf{f}(x-x,y) \approx \mathsf{T}$. This results in the following equational system:

$$x-\mathsf{0} \approx x \quad \mathsf{0}-x \approx \mathsf{0} \quad \mathsf{s}(x)-\mathsf{s}(y) \approx x-y \quad \mathsf{f}(\mathsf{s}(x),x) \approx \mathsf{F} \quad \mathsf{f}(x-x,y) \approx \mathsf{T}$$

Now maximal ordered completion builds up the finite ground-complete presentation $\mathcal{R}$ of the approximated equational system:

$$x-\mathsf{0} \to x \quad \mathsf{0}-x \to \mathsf{0} \quad \mathsf{s}(x)-\mathsf{s}(y) \to x-y \quad \mathsf{f}(\mathsf{0},y) \to \mathsf{T} \quad \mathsf{f}(\mathsf{s}(x),x) \to \mathsf{F} \quad \mathsf{f}(x-x,y) \to \mathsf{T}$$

Since $\mathsf{T}{\downarrow_{\mathcal{R}}} \neq \mathsf{F}{\downarrow_{\mathcal{R}}}$ holds, infeasibility of the conversion $x-x \leftrightarrow^* \mathsf{s}(x)$ is concluded. Version 0.2 of Moca supports the generalized split-if encoding [3] and *inlining* for conditional rewrite rules [4].

References

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