# CO3

## a COnverter for proving COnfluence of COnditional term rewriting systems

Ver. 1.2

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# Target and Main Function (the same as last year)

Convert a normal 1-CTRS to a TRS by using

- the simultaneous unraveling $\mathbb{U}$ [Marchiori, 96][Ohlebusch, 02][Gmeiner et al, 13]
- the SR transformation $\mathbb{SR}$ [Șerbănuță & Roșu, 06]
  - if the input CTRS is constructor-based, then the special bracket symbol and its rewrite rules are not introduced, i.e., the result is the same as by [Antoy et al, 03]

**Theorem (theoretical background)**

*$\mathcal{R}$ is confluent if $\mathcal{R}$ is weakly left-linear (WLL) and*

- *$\mathbb{U}(\mathcal{R})$ is confluent,* *[Gmeiner et al, 13]*
  *or*
- *$\mathbb{SR}(\mathcal{R})$ is confluent* *[Nishida et al, 14]*

# Difference from Last Year

- Transform condition $c(t_1, \ldots, t_n) = c(u_1, \ldots, u_n)$ into $t_1 = u_1, \ldots, t_n = u_n$ as much as possible, where c is a constructor
- Drop rules with an infeasible condition $s = t$ such that
  - $REN(CAP(s))$ is not unifiable with $t$,
    e.g., 264.trs $= \{f(x, x) \to a \Leftarrow g(x) = b\}$ is transformed into $\emptyset$
    or
  - any instance of $\mathtt{root}(s)$ is not reachable to any instance of $\mathtt{root}(t)$ w.r.t. $\{\mathtt{root}(l) \to \mathtt{root}(r) \mid l \to r \Leftarrow c \in \mathcal{R}\}$
    - $\mathtt{root}(r)$ may be a variable

    e.g., 271.trs $= \{\ldots, r(x) \to r(h(x)) \Leftarrow s(x) = 0, s(x) \to 1\}$ is transformed into a TRS $\{p(q(x)) \to p(r(x)), q(h(x)) \to r(x), s(x) \to 1\}$
- Drop all $f$-rules such that each $f$-rule $l \to r \Leftarrow c$ has a condition $f(s_1, \ldots, s_n) = t$ in $c$ such that $REN(f(CAP(s_1), \ldots, CAP(s_n))$ is not unifiable with $t$ (i.e., all $f$-rules are infeasible)

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