CoLL: A Confluence Tool Based on Commutation*

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CoLL (ver. 0.1) is a tool for automatically proving confluence and **co**mmutation of (ordinary) **l**eft-**l**inear term rewrite systems (TRSs). The tool, written in OCaml, is freely available at

`http://www.jaist.ac.jp/~s1310032/coll/`

The typical usage is as follows: `coll <file>` proves confluence and `coll <file> <file>` proves commutation. Here the input files are written in the standard WST format. The tool outputs `YES` if confluence (commutation) of the input TRS is proved and `MAYBE` if the tool does not reach any conclusion. The following abstract criteria constitute the kernel of the tool.

Theorem 1 ([4, 5])**.** *ARSs $\mathcal{A} = \langle A, \{\to_\alpha\}_{\alpha\in I}\rangle$ and $\mathcal{B} = \langle A, \{\to_\beta\}_{\beta\in J}\rangle$ commute if $\to_a$ and $\to_b$ commute for all $a \in I$ and $b \in J$.*

Theorem 2. *An ARS $\mathcal{A}$ is confluent if $\mathcal{A}$ and $\mathcal{A}$ commute.*

As listed below, CoLL implements the four commutation criteria: A variant of Knuth-Bendix' criterion, the development closedness theorem [2, Proposition 2], a criterion based on [3], and the extended rule labeling [1, Theorem 5.4].

Theorem 3. *Left-linear TRSs $\mathcal{R}$ and $\mathcal{S}$ commute if $\mathcal{R}\cup\mathcal{S}$ terminating and $\mathcal{R}$ and $\mathcal{S}$ are locally commuting.*

Theorem 4. *Left-linear TRSs $\mathcal{R}$ and $\mathcal{S}$ commute if $\xleftarrow[\mathcal{R}]{}^{>\epsilon} \rtimes \xrightarrow[\mathcal{S}]{} \subseteq \xrightarrow[\mathcal{S}]{}\circ\xleftarrow[\mathcal{R}]{}$ and $\xleftarrow[\mathcal{R}]{} \ltimes \xrightarrow[\mathcal{S}]{} \subseteq \xrightarrow[\mathcal{S}]{}^{*} \cdot \xleftarrow[\mathcal{R}]{}\circ$.*

Theorem 5. *Left-linear TRSs $\mathcal{R}$ and $\mathcal{S}$ commute if $\mathcal{R}$ is terminating, $\mathcal{S}$ is non-erasing, and $\left(\xleftarrow[\mathcal{R}]{} \ltimes \xrightarrow[\mathcal{S}]{}\right) \cup \left(\xleftarrow[\mathcal{R}]{} \rtimes \xrightarrow[\mathcal{S}]{}\right) \subseteq \xrightarrow[\mathcal{S}]{}^{*} \cdot \xleftarrow[\mathcal{R}]{}^{+}$ holds.*

Theorem 6. *A TRS $\mathcal{R}$ is confluent if every critical peak is decreasing with respect to the weight-based rule labeling heuristic.*

CoLL employs MaxSAT to find suitable commuting subsystems for Theorem 1. Termination is proved by the lexicographic path order, and local commutation is tested by the next inclusion:

$$\left(\xleftarrow[\mathcal{R}]{} \ltimes \xrightarrow[\mathcal{S}]{}\right) \cup \left(\xleftarrow[\mathcal{R}]{} \rtimes \xrightarrow[\mathcal{S}]{}\right) \;\subseteq\; \xrightarrow[\mathcal{S}]{}^{\leqslant 4} \cdot \xleftarrow[\mathcal{R}]{}^{\leqslant 4}$$

References

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