



12th Confluence Competition

Raúl Gutiérrez **Aart Middeldorp** Naoki Nishida Teppei Saito

<http://project-coco.uibk.ac.at/2023>

Outline

- 1. Acknowledgements**
- 2. History**
- 3. 2023**
- 4. Awards**
- 5. Outlook**

Acknowledgements

- CoCo 2023 tool authors

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- Fabian Mitterwallner

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- Jonas Schöpf

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- IWC 2023 chairs

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 - Mirai Ikebuchi
 - Nick Smallbone

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 - Nick Smallbone
- Aaron Stump

Outline

1. Acknowledgements

2. History

3. 2023

4. Awards

5. Outlook

2012

TRS





CPF

<input checked="" type="checkbox"/> ACP	✓											
<input checked="" type="checkbox"/> CSI	✓	✓										
<input checked="" type="checkbox"/> Saigawa	✓											
<input checked="" type="checkbox"/> CeTA		✓										

2013

TRS

CPF








 ACP	✓																			
 CSI	✓	✓																		
 Saigawa	✓																			
 CeTA		✓																		

2014

TRS

CPF

CTRS

 ACP	✓	✓												
 CSI	✓	✓												
 Saigawa	✓													
 CeTA		✓												
 CoLL	✓													
 ConCon		✓	✓											
 CO3			✓											

2015

TRS

CPF

CTRS

HRS













GCR

NRS

	TRS	CPF	CTRS	HRS	GCR	NRS									
● ACP	✓	✓													
≡ CSI	✓	✓													
≡ CeTA		✓													
● CoLL-Saigawa	✓														
≡ ConCon		✓	✓												
⊖ CO3			✓												
≡ CoScart			✓												
● ACPH				✓											
≡ CSI ^{ho}				✓											
● AGCP					✓										
● NoCo						✓									

2016

TRS CPF-TRS CTRS HRS GCR NRS UN CPF-CTRS

	TRS	CPF-TRS	CTRS	HRS	GCR	NRS	UN	CPF-CTRS				
 ACP	✓	✓										
 CSI	✓	✓					✓					
 CeTA		✓						✓				
 CoLL-Saigawa	✓											
 ConCon			✓					✓				
 CO3			✓									
 CoScart			✓									
 ACPH				✓								
 CSI^ho				✓								
 AGCP					✓							
 Nrbox						✓						
 FORT					✓		✓					











2017

TRS CPF-TRS CTRS HRS GCR NFP UNR CPF-CTRS UNC

	TRS	CPF-TRS	CTRS	HRS	GCR	NFP	UNR	CPF-CTRS	UNC			
● ACP	✓	✓										
≡ CSI	✓	✓				✓	✓		✓			
≡ CeTA		✓						✓				
● CoLL-Saigawa	✓											
≡ ConCon			✓					✓				
⊖ CO3			✓									
● ACPH				✓								
≡ CSI ^{ho}				✓								
● AGCP					✓							
≡ FORT					✓	✓	✓		✓			
● SOL				✓								
















2018

TRS CPF-TRS CTRS HRS GCR NFP UNR CPF-CTRS UNC

	TRS	CPF-TRS	CTRS	HRS	GCR	NFP	UNR	CPF-CTRS	UNC			
 ACP	✓	✓	✓						✓			
 CSI	✓	✓				✓	✓		✓			
 CeTA		✓						✓				
 CoLL-Saigawa	✓											
 ConCon			✓					✓				
 CO3			✓									
 CSI^ho				✓								
 AGCP					✓							
 FORT					✓	✓	✓		✓			
 SOL				✓								















2019

TRS CPF-TRS CTRS HRS GCR NFP UNR CPF-CTRS UNC COM INF SRS













	TRS	CPF-TRS	CTRS	HRS	GCR	NFP	UNR	CPF-CTRS	UNC	COM	INF	SRS
 ACP	✓	✓	✓						✓	✓		✓
 CSI	✓	✓				✓	✓		✓			✓
 CoLL										✓		
 CeTA		✓						✓				
 CoLL-Saigawa	✓											✓
 ConCon			✓					✓			✓	
 CO3			✓								✓	
 infChecker											✓	
 maedmax											✓	
 CSI^ho				✓								
 AGCP					✓							
 Moca											✓	
 FORT					✓	✓	✓		✓	✓		
 noko-leipzig												✓
 nonreach											✓	

2020














TRS CPF-TRS CTRS HRS GCR NFP UNR CPF-CTRS UNC COM INF SRS


	TRS	CPF-TRS	CTRS	HRS	GCR	NFP	UNR	CPF-CTRS	UNC	COM	INF	SRS
 ACP	✓	✓	✓						✓	✓		✓
 CSI	✓	✓				✓	✓		✓			✓
 CoLL										✓		
 CeTA		✓						✓				
 CoLL-Saigawa	✓											✓
 ConCon			✓					✓			✓	
 CO3			✓								✓	
 infChecker											✓	
 CSI ^{ho}				✓								
 AGCP					✓							
 Moca											✓	
 FORT-h					✓	✓	✓		✓	✓		
 SOL				✓								
 nonreach												✓

2021

	TRS	CPF-TRS	CTRS		GCR	NFP	UNR		UNC	COM	INF	SRS
 ACP	✓	✓	✓						✓	✓		✓
 CSI	✓	✓				✓	✓		✓			✓
 CoLL										✓		
 CeTA		✓										
 CoLL-Saigawa	✓											✓
 CO3			✓								✓	
 infChecker											✓	
 FORTify	✓				✓		✓		✓	✓		
 CONFident	✓		✓									✓
 AGCP					✓							
 NaTT											✓	
 FORT-h					✓	✓	✓		✓	✓		

2022

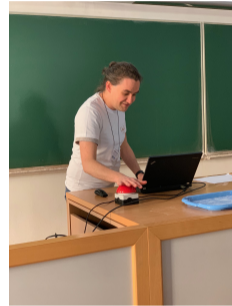
	TRS	CPF-TRS	CTRS		GCR	NFP	UNR	CSR	UNC	COM	INF	SRS
 ACP	✓	✓	✓						✓	✓		✓
 CSI	✓	✓				✓	✓		✓			✓
 CoLL										✓		
 CeTA		✓										
 Hakusan	✓											✓
 Toma											✓	
 CO3			✓								✓	
 infChecker											✓	
 FORTify		✓			✓	✓	✓		✓	✓		
 CONFident	✓		✓					✓				✓
 AGCP					✓							
 NaTT											✓	
 FORT-h		✓			✓	✓	✓		✓	✓		


- CoCo is powered by StarExec 

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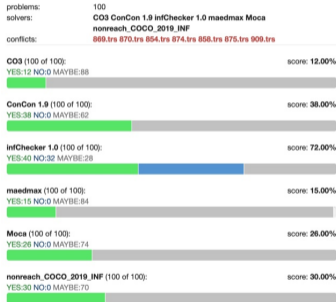


- **exciting** to watch

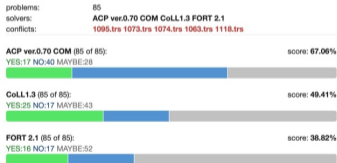


- CoCo is powered by StarExec 
- **exciting** to watch, partly due to real-time **yes/no conflicts**

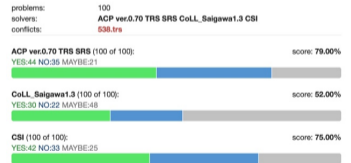
INF



COM



TRS



final slide CoCo 2022 presentation

- CoCo 2023 during IWC 2023 in Obergurgl







final slide CoCo 2022 presentation

- CoCo 2023 during IWC 2023 in Obergurgl
- no separate CPF categories: tool + certifier runs in categories in which tool participates

final slide CoCo 2022 presentation

- CoCo 2023 during IWC 2023 in Obergurgl
- no separate CPF categories: tool + certifier runs in categories in which tool participates
- revive previous categories?
 - HRS
 - CPF-CTRS
 - NRS

final slide CoCo 2022 presentation

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- no separate CPF categories: tool + certifier runs in categories in which tool participates
- revive previous categories?
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Outline

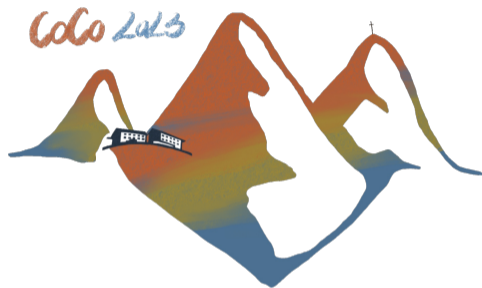
1. Acknowledgements

2. History

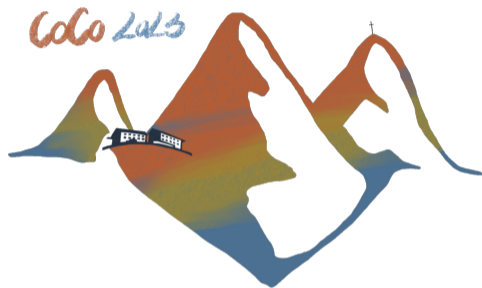
3. 2023

4. Awards

5. Outlook
















<http://cocograph.uibk.ac.at/2023.html>


















T shirt 20 €

<http://cocograph.uibk.ac.at/2023.html>

2022

	TRS	CPF-TRS	CTRS		GCR	NFP	UNR	CSR	UNC	COM	INF	SRS
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 Hakusan	✓											✓
 Toma											✓	
 CO3			✓								✓	
 infChecker											✓	
 FORTify		✓			✓	✓	✓		✓	✓		
 CONFident	✓		✓					✓				✓
 AGCP					✓							
 NaTT											✓	
 FORT-h		✓			✓	✓	✓		✓	✓		

2023

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 CoLL										✓		
 CeTA	✓									✓		✓
 Hakusan	✓											✓
 Toma											✓	
 CO3			✓								✓	
 infChecker											✓	
 FORTify	✓				✓	✓	✓		✓	✓		
 CONFident	✓		✓					✓				✓
 AGCP					✓							
 NaTT											✓	
 FORT-h	✓				✓	✓	✓		✓	✓		
 ConfCSR								✓				
 nonreach											✓	

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3. 2023

Categories

Rules

Live View

4. Awards

5. Outlook

Categories

TRS	confluence of first-order term rewrite systems
CTRS	confluence of first-order conditional term rewrite systems
HRS	confluence of higher-order rewrite systems
GCR	ground-confluence of many-sorted first-order rewrite systems
NFP	normal form property of first-order rewrite systems
UNR	unique normal forms wrt reduction of first-order rewrite systems
UNC	unique normal forms wrt conversion of first-order rewrite systems
COM	commutation of first-order rewrite systems
INF	infeasibility
SRS	confluence of string rewrite systems
CSR	confluence of context-sensitive rewriting

Categories

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Competition Rules

- **Scoring**
 - 100 random problems per category, using seed digits provided by panel members
 - tools output YES, NO or MAYBE on first line followed by proof
 - separate rankings for YES and NO and combined YES/NO answers
 - winning tools of 2022 participate as demonstration tools in 2023

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 - tools output YES, NO or MAYBE on first line followed by proof
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 - winning tools of 2022 participate as demonstration tools in 2023
- **Secret Problems**
 - guaranteed to be selected
 - at most two problems per category per tool

Secret Problems

4 secret problems submitted to CoCo 2022:

- 2 by Christina Kohl (CeTA)

$$f(g(x)) \rightarrow h(x, x)$$

$$h(x, y) \rightarrow h(g(x), g(y))$$

$$f(x) \rightarrow h(x, x)$$

$$g(a) \rightarrow b$$

$$b \rightarrow a$$

$$g(x) \rightarrow x$$

$$a \rightarrow b$$

$$f(x_1, g(x_2)) \rightarrow f(x_1, g(x_1))$$

$$f(g(y_1), y_2) \rightarrow f(g(y_1), g(y_1))$$

$$g(a) \rightarrow g(b)$$

$$b \rightarrow a$$

- 2 by Miguel Vítores and Salvador Lucas (CONFident)

$$g(g(g(c, d), g(b, e)), b) \rightarrow a$$

$$g(a, y) \rightarrow g(b, b)$$

$$g(x, y) \rightarrow g(y, x)$$

$$f(h(f(f(a, a), h(a))), g(f(x, g(b)))) \rightarrow c$$

$$g(g(a)) \rightarrow f(h(g(f(c, c))), f(f(g(c), a), g(f(a, a))))$$

$$c \rightarrow c$$

$$c \rightarrow f(a, h(b))$$

$$f(x, y) \rightarrow f(y, x)$$

Secret Problems

0 secret problems submitted to CoCo 2023

Competition Rules

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Problem Selection

Example: COPS queries for TRS category

① `limit:100,068 1..1655 trs !srs !duplicate {!confluent !non_confluent}`

② `limit:75,068 1..1655 trs !srs !duplicate {!confluent !non_confluent}`

Problem Selection

Example: COPS queries for TRS category

① `limit:100,068 1..1655 trs !srs !duplicate {!confluent !non_confluent}`

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- seed digits of panel members

Problem Selection

Example: COPS queries for TRS category

① `limit:100,068 1..1655 trs !srs !duplicate {!confluent !non_confluent}`

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- seed digits of panel members
- exclude SRSs

Problem Selection

Example: COPS queries for TRS category

① `limit:100,068 1..1655 trs !srs !duplicate {!confluent !non_confluent}`

② `limit:75,068 1..1655 trs !srs !duplicate !{!confluent !non_confluent}`

- seed digits of panel members
- exclude SRSs, duplicates

Problem Selection

Example: COPS queries for TRS category

① `limit:100,068 1..1655 trs !srs !duplicate {!confluent !non_confluent}`

② `limit:75,068 1..1655 trs !srs !duplicate {!confluent !non_confluent}`

- seed digits of panel members
- exclude SRSs, duplicates and TRSs whose confluence status is known

Problem Selection

Example: COPS queries for TRS category

① `limit:100,068 1..1655 trs !srs !duplicate {!confluent !non_confluent}`
returns 25 problems whose status is unknown

② `limit:75,068 1..1655 trs !srs !duplicate !{!confluent !non_confluent}`

- seed digits of panel members
- exclude SRSs, duplicates and TRSs whose confluence status is known

Problem Selection

Example: COPS queries for TRS category

- 1 `limit:100,068 1..1655 trs !srs !duplicate {!confluent !non_confluent}`
returns 25 problems whose status is unknown
 - 2 `limit:75,068 1..1655 trs !srs !duplicate !{!confluent !non_confluent}`
returns 75 random problems whose status is known
- seed digits of panel members
 - exclude SRSs, duplicates and TRSs whose confluence status is known

Competition Rules

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- Secret Problems
 - guaranteed to be selected
 - at most two problems per category per tool
- **Incorrect Results**
 - tools with incorrect results (observed during live competition due to YES/NO conflict, or communicated afterwards by tool authors to SC) are excluded from results table

Competition Rules

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 - 100 random problems per category, using seed digits provided by panel members
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 - guaranteed to be selected
 - at most two problems per category per tool
- **Incorrect Results**
 - tools with incorrect results (observed during live competition due to YES/NO conflict, or communicated afterwards by tool authors to SC) are excluded from results table
 - (corrected) tools are available from **CoCoWeb** for testing



Tools

2021

CTRS

SRS

TRS

2020

2019

2018

2017

2016

2015

2014

2013

2012

Enter a **rewrite system**, upload a file or import a Cop:

```
1 (CONDITIONTYPE ORIENTED)
2 (VAR x)
3 (RULES
4   f(x) -> g(a) | g(x) == x
5   f(x) -> g(b)
6 )
7 (COMMENT
8   submitted by: Raul Gutierrez
9   secret problem 2021
10  category: CTRS
11 )
```

property:
 timeout: submit this problem to



Tools

2021

CTRS

ACP

CO3

CONFident

SRS

TRS

2020

2019

2018

2017

2016

2015

2014

2013

2012

Enter a **rewrite system**, upload a file or import a Cop:

```

1 (CONDITIONTYPE ORIENTED)
2 (VAR x)
3 (RULES
4   f(x) -> g(a) | g(x) == x
5   f(x) -> g(b)
6 )
7 (COMMENT
8   submitted by: Raul Gutierrez
9   secret problem 2021
10  category: CTRS
11 )

```

 property:

 timeout:

Results

Took 0.03s

MAYBE

(ignored inputs)COMMENT submitted by: Raul Gutierrez secret problem

2021 category: CTRS

Conditional Rewrite Rules:

```

[ f(?x) -> g(a) | g(?x) == ?x,
  f(?x) -> g(b) ]

```

Check whether all rules are type 3

OK

Check whether the input is deterministic

format: [all](#) [trs](#) [etrs](#) [ctrs](#) [cstrs](#) [csctrs](#) [hrs](#) [mstrs](#) [+](#)search: 1 problems matched. [DOWNLOAD .zip](#) with tag & bib files and copsPREV [1](#) » NEXTorder: [desc](#) [asc](#)COPS: [inlined](#) [plain](#)[1655.trs](#)[CoCoWeb](#)

```
(VAR x1 x2 y1 y2)
(RULES
  f(x1,g(x2)) -> f(x1, g(x1))
  f(g(y1),y2) -> f(g(y1), g(y1))
  g(a) -> g(b)
  b -> a
)
(COMMENT
submitted by: Christina Kohl
secret problem 2022
category: TRS
)
```

format:

[trs](#) [+](#)tags: [confluent](#) [left_linear](#) [locally_confluent](#) [nfp](#) [non_ground](#) [non_linear](#) [non_orthogonal](#) [non_right_ground](#) [non_right_linear](#) [non_terminating](#) [non_weakly_orthogonal](#) [unc](#) [unr](#) [+](#)

versions:

[cops2022](#) [+](#)PREV [1](#) » NEXTorder: [desc](#) [asc](#)COPS: [inlined](#) [plain](#)

confluence problems database (COPS) consists of 1655 problems

CoCo 2023 Live View

<http://cocograph.uibk.ac.at/2023.html>

2022 Results



2022 Results



- previous winners: ACP (2012 2013 2014 2015 2019)
CSI (2015 2016 2017 2018 2020 2021)

2022 Results



- previous winners: ACP (2012 2013 2014 2015 2019)
CSI (2015 2016 2017 2018 2020 2021)
- 2023 participants: ACP ACP+CeTA CONFident CSI CSI+CeTA FORT-h
FORT-h + FORTify Hakusan **Hakusan + CeTA**



CSI 1.2.7

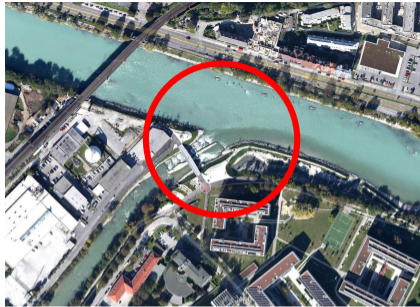
Fabian Mitterwallner

Aart Middeldorp

University of Innsbruck

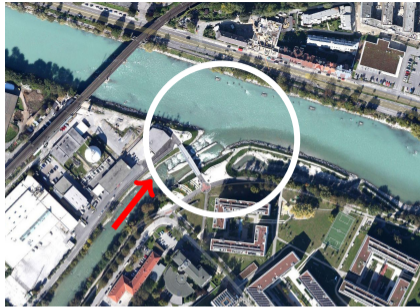


[Google Maps]



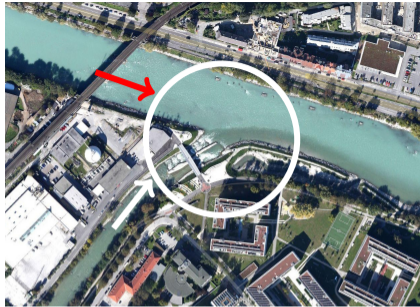
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C



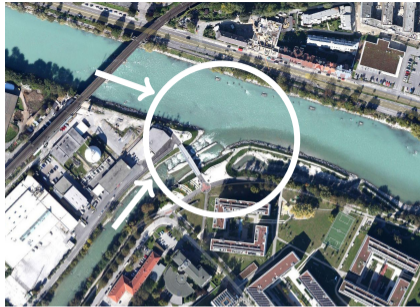
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CS



[Google Maps]

CSI



[Google Maps]

CSI 1.2.7

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- ▶ SRS TRS UNC UNR NFP CPF-TRS (with CeTA)

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CoCo 2022 Categories

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New Features

- ▶ CSI can produce certificates for left-linear **almost** development-closed rewrite systems which are validated by **CeTA 2.45** (Kohl and Middeldorp, ITP 2023)

CONFident at CoCo 2023

Miguel Vítors¹ Raúl Gutiérrez² Salvador Lucas¹

OBERGURGL, AUGUST 23RD AND 24TH, 2023

¹Valencian Research Institute for Artificial Intelligence
Universitat Politècnica de València
Spain

²Universidad Politécnica de Madrid
Spain

- CONFident is a tool for checking **(non-)confluence** of Generalized Term Rewriting Systems (GTRSs).
- A GTRS is a tuple $\mathcal{R} = (\Omega, \mu, H, R)$, where:
 - $\Omega = (\mathcal{F}, \Pi)$ is a signature with predicates.
 - $\mu \in M_{\mathcal{F}}$.
 - H is a set of auxiliary clauses (H is used to model the semantics of conditions).
 - R is a set of rewrite rules $\ell \rightarrow r \Leftarrow c$.

- The tool is available here:
`http://zenon.dsic.upv.es/confident/`.
- It is written in Haskell and implements the **Confluence Framework**, a divide-and-conquer squema where different techniques are applied recursively using a pre-defined strategy.
- In the Confluence Framework, we consider two types of problems:
 - **Confluence Problems**, that encapsulate the GTRSs whose confluence is tested.
 - **Joinability Problems**, that are used to prove or disprove joinability.

- The **proof strategy** used in CONFident is as follows:
 - 1 it tries a simplification processor to simplify the input system or its rules;
 - 2 it tries to decompose the problem using a modularity processor;
 - 3 at this point, there exists an alternative option that potentially involves the use of transformational processors (P_{CanCR} , P_U or P_{Uconf});
 - 4 for each branch, it extracts their extended critical pairs, thereby generating joinability problems (termination checks can also be applied);
 - 5 finally, it attempts joinability checks on each joinability problem.

- GLV23** R. Gutiérrez, S. Lucas and M. Vítores. Proving Confluence in the Confluence Framework with CONFident. CoRR abs/2306.16330, 2023.
- LVG22** S. Lucas, M. Vítores and R. Gutiérrez. Proving and disproving confluence of context-sensitive rewriting. J. Log. Algebraic Methods Program. 126: 100749, 2022.
- GL22** R. Gutiérrez, M. Vítores and S. Lucas. Confluence Framework: Proving Confluence with CONFident. In Proc. of LOPSTR'2022: 24-43, 2022.

2022 Results



- Hakusan had most YES answers

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Hakusan+CeTA

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- **noko-leipzig** produced most NO answers in 2019



v0.8 (Shintani & Hirokawa, JAIST)

confluence tool for **left-linear** TRSs, supporting:



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Theorem (reduction method)

$CR(\mathcal{R}) \iff CR(\mathcal{C})$ if \mathcal{R} is left-linear, $\mathcal{C} \subseteq \mathcal{R}$, $PCP(\mathcal{R}) \subseteq \leftrightarrow_{\mathcal{C}}^*$, and $\mathcal{R}|_{\mathcal{C}} \subseteq \rightarrow_{\mathcal{C}}^*$

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where $\mathcal{R}|_{\mathcal{C}} = \{\ell \rightarrow r \in \mathcal{R} \mid \mathcal{F}un(\ell) \subseteq \mathcal{F}un(\mathcal{C})\}$

using **reduction method**, we show confluence of left-linear TRS \mathcal{R} :

$$\begin{array}{lll} 1: & x + 0 \rightarrow x & 3: \quad 0 + y \rightarrow y & 5: \quad s(x) + y \rightarrow s(x + y) \\ 2: & x \times 0 \rightarrow 0 & 4: \quad s(x) \times 0 \rightarrow 0 & 6: \quad s(x) \times y \rightarrow (x \times y) + y \end{array}$$

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 \swarrow \neq & \swarrow \neq & \swarrow \neq & \swarrow \neq \\
 0 & 0 & s(x) & s(x + 0) \\
 = & = & & \\
 0 & 0 & & \\
 & & \xrightarrow{1} & \\
 & & & s(x)
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 & & \xrightarrow{1} & & \xrightarrow{1}
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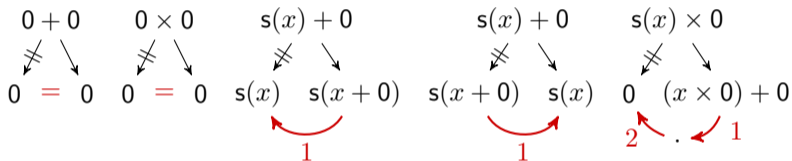
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 0 = 0 & 0 = 0 & s(x) & s(x + 0) & s(x + 0) & s(x) & 0 & (x \times 0) + 0 \\
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$0 + 0$ 0×0 $s(x) + 0$ $s(x) + 0$ $s(x) \times 0$ $s(x) \times 0$
 $\swarrow \not\equiv \searrow$ $\swarrow \not\equiv \searrow$ $\swarrow \not\equiv \searrow$ $\swarrow \not\equiv \searrow$ $\swarrow \not\equiv \searrow$ $\swarrow \not\equiv \searrow$
 $0 = 0$ $0 = 0$ $s(x)$ $s(x + 0)$ $s(x + 0)$ $s(x)$ 0 $(x \times 0) + 0$ $(x \times 0) + 0$ 0
↷ **1** ↷ **1** ↷ **2** ↷ **1**

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$$\begin{array}{c}
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 \swarrow \quad \searrow \\
 0 = 0
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 \quad
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and subsystem over $\mathcal{F}un(\{1, 2\}) = \{0, +, \times\}$

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The diagram illustrates confluence proofs for three rules:

- Rule 1:** $0 + 0 \rightarrow 0$. Two paths from $0 + 0$ lead to $0 = 0$.
- Rule 2:** $0 \times 0 \rightarrow 0$. Two paths from 0×0 lead to $0 = 0$.
- Rule 3:** $s(x) + 0 \rightarrow s(x)$. Two paths from $s(x) + 0$ lead to $s(x)$. A red arrow labeled '1' indicates confluence.
- Rule 4:** $s(x) \times 0 \rightarrow 0$. Two paths from $s(x) \times 0$ lead to 0 . A red arrow labeled '1' indicates confluence.
- Rule 5:** $s(x) + y \rightarrow s(x + y)$. Two paths from $s(x) + y$ lead to 0 and $(x \times 0) + 0$. Red arrows labeled '2' and '1' indicate confluence.
- Rule 6:** $s(x) \times y \rightarrow (x \times y) + y$. Two paths from $s(x) \times 0$ lead to $(x \times 0) + 0$ and 0 . Red arrows labeled '1' and '2' indicate confluence.

and subsystem over $\mathcal{F}un(\{1, 2\}) = \{0, +, \times\}$ is $\{1, 2, 3\}$

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1 $CR(\mathcal{R}) \iff CR(\{1, 2, 3\})$ because

$0 + 0 \rightarrow 0 = 0$ (no confluence arrows)
 $0 \times 0 \rightarrow 0 = 0$ (no confluence arrows)
 $s(x) + 0 \rightarrow s(x) \rightarrow s(x + 0)$ (confluence arrow labeled 1)
 $s(x) + 0 \rightarrow s(x + 0) \rightarrow s(x)$ (confluence arrow labeled 1)
 $s(x) \times 0 \rightarrow 0 \rightarrow (x \times 0) + 0$ (confluence arrows labeled 2 and 1)
 $s(x) \times 0 \rightarrow (x \times 0) + 0 \rightarrow 0$ (confluence arrows labeled 1 and 2)

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① $CR(\mathcal{R}) \iff CR(\{1, 2, 3\})$ because

$0 + 0$ and 0×0 both reduce to $0 = 0$.
 $s(x) + 0$ and $s(x) \times 0$ both reduce to $s(x)$ and 0 respectively. A red arrow labeled '1' connects $s(x)$ and 0 .
 $s(x) + 0$ and $s(x) \times 0$ both reduce to $s(x + 0)$ and $s(x)$ respectively. A red arrow labeled '1' connects $s(x + 0)$ and $s(x)$.
 $s(x) \times 0$ and $s(x) \times 0$ both reduce to 0 and $(x \times 0) + 0$ respectively. Red arrows labeled '2' and '1' connect 0 and $(x \times 0) + 0$.
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and subsystem over $\mathcal{F}un(\{1, 2\}) = \{0, +, \times\}$ is $\{1, 2, 3\}$

② $CR(\{1, 2, 3\}) \iff CR(\emptyset)$

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and subsystem over $\mathcal{F}un(\emptyset) = \emptyset$ is \emptyset

using **reduction method**, we show confluence of left-linear TRS \mathcal{R} :

$$\begin{array}{lll}
 1: & x + 0 \rightarrow x & 3: & 0 + y \rightarrow y & 5: & s(x) + y \rightarrow s(x + y) \\
 2: & x \times 0 \rightarrow 0 & 4: & s(x) \times 0 \rightarrow 0 & 6: & s(x) \times y \rightarrow (x \times y) + y
 \end{array}$$

1 $CR(\mathcal{R}) \iff CR(\{1, 2, 3\})$ because

$$\begin{array}{cccccc}
 \begin{array}{c} 0 + 0 \\ \swarrow \neq \searrow \\ 0 = 0 \end{array} &
 \begin{array}{c} 0 \times 0 \\ \swarrow \neq \searrow \\ 0 = 0 \end{array} &
 \begin{array}{c} s(x) + 0 \\ \swarrow \neq \searrow \\ s(x) \quad s(x + 0) \\ \xrightarrow{1} \end{array} &
 \begin{array}{c} s(x) + 0 \\ \swarrow \neq \searrow \\ s(x + 0) \quad s(x) \\ \xrightarrow{1} \end{array} &
 \begin{array}{c} s(x) \times 0 \\ \swarrow \neq \searrow \\ 0 \quad (x \times 0) + 0 \\ \xrightarrow{2} \quad \xrightarrow{1} \end{array} &
 \begin{array}{c} s(x) \times 0 \\ \swarrow \neq \searrow \\ (x \times 0) + 0 \quad 0 \\ \xrightarrow{1} \quad \xrightarrow{2} \end{array}
 \end{array}$$

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$0 + 0$ \searrow $0 = 0$ \swarrow
 0×0 \searrow $0 = 0$ \swarrow
 $s(x) + 0$ \searrow $s(x)$ \swarrow $s(x + 0)$ $\xrightarrow{1}$
 $s(x) + 0$ \searrow $s(x + 0)$ \swarrow $s(x)$ $\xrightarrow{1}$
 $s(x) \times 0$ \searrow 0 \swarrow $(x \times 0) + 0$ $\xrightarrow{2}$ $\xrightarrow{1}$
 $s(x) \times 0$ \searrow $(x \times 0) + 0$ \swarrow 0 $\xrightarrow{1}$ $\xrightarrow{2}$

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$0 + 0$ \searrow $0 = 0$ \swarrow

and subsystem over $\mathcal{F}\text{un}(\emptyset) = \emptyset$ is \emptyset

3 $\text{CR}(\emptyset)$ is trivial,

using **reduction method**, we show confluence of left-linear TRS \mathcal{R} :

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2 $\text{CR}(\{1, 2, 3\}) \iff \text{CR}(\emptyset)$ because

and subsystem over $\mathcal{F}\text{un}(\emptyset) = \emptyset$ is \emptyset

3 $\text{CR}(\emptyset)$ is trivial, hence $\text{CR}(\mathcal{R})$



CoCo 2023 Participant: CeTA 2.45

René Thiemann, Christina Kohl, and Dohan Kim

CeTA 2.45



- CeTA: certifier of various properties, verified in Isabelle/HOL
- developed by Computational Logic Group in Innsbruck
- several confluence techniques supported, see a complete list at:
<http://cl-informatik.uibk.ac.at/software/ceta/>
- new techniques in CeTA 2.45 in comparison to 2022
 - almost development closed TRSs are confluent (van Oostrom)
 - techniques relying on parallel critical pairs (criteria of Gramlich and Toyama, rule labeling, compositional versions of Shintani and Hirokawa)
 - support for commutation (almost development closed, (compositional) parallel critical pair criteria)
 - support for non-commutation (non-joinable peaks)

2022 Results



2022 Results



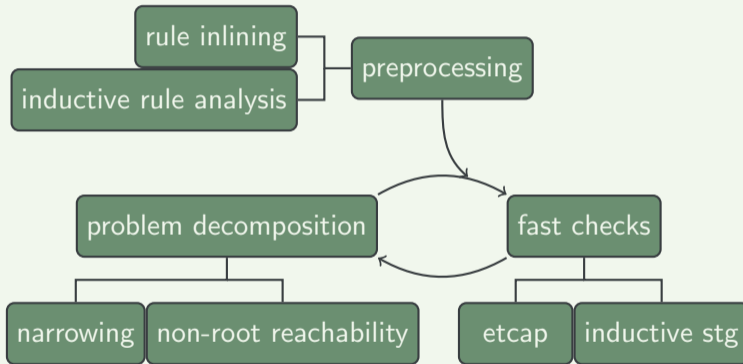
- previous winner: infChecker (2019 2020 2021)

2022 Results



- previous winner: infChecker (2019 2020 2021)
- 2023 participants: CO3 infChecker NaTT nonreach **nonreach+CeTA** Toma

nonreach 1.3



Features

- open source <https://bitbucket.org/fmessner/nonreach/>
- fast infeasibility checks based on tcap and the inductive symbol transition graph
- combined with decomposition into easier subproblems
- simple interface
- Yes and No results
- can output certificates

CoCo 2023

- INF category
- nonreach and nonreach+CeTA

Toma

Teppei Saito Nao Hirokawa

JAIST

IWC 2023

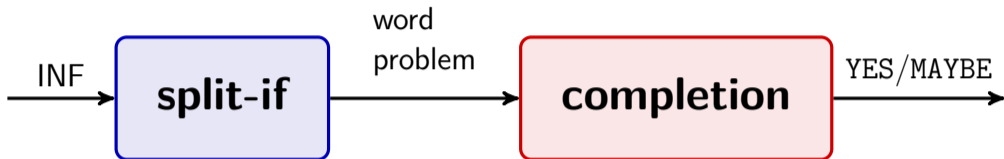
Toma: **T**heorem prover based on **o**rdered **m**aximal completion



equational theorem prover **Toma**

<https://www.jaist.ac.jp/project/maxcomp/>

Architecture



- 1 INF problem is transformed into word problem by *split-if* encoding (Claessen and Smallbone, 2018).
- 2 Word problem is solved by new variant of maximal ordered completion:
 - maximal ordered completion (Winkler and Moser, 2018) plus
 - maximal completion with inter-reduction (Hirokawa, 2021)

Witness of infeasibility is (ground-complete) term rewrite system.

NaTT in CoCo 2023

Akihisa Yamada @ AIST

NaTT in CoCo

- NaTT a **termination prover** for **plain** term rewriting
- In CoCo 2021:
 - participated **INF** category with an easy checker [Sternagel & Yamada '19]
 - ignores conditions
 - weak but fast
- In CoCo 2022:
 - **coWPO** for INF [IJCAR '22]
 - don't ignore conditions
 - not too weak, but slow
- In CoCo 2023:
 - nothing changed...



UNIVERSITAT
POLITÈCNICA
DE VALÈNCIA



UNIVERSIDAD
POLITÈCNICA
DE MADRID

infChecker at CoCo 2023

Raúl Gutiérrez¹ Salvador Lucas² Miguel Víttores²

OBERGURGL, AUGUST 23RD AND 24TH, 2023

¹Universidad Politécnica de Madrid
Spain

²Valencian Research Institute for Artificial Intelligence
Universitat Politècnica de València
Spain

Description

- This year, our participation involves utilizing the same tool employed in the previous year.
- infChecker is a tool for checking **(in)feasibility of goals** $\mathcal{G} = \{F_i\}_{i=1}^m$, where $F_i = (s_{ij} \bowtie_{ij} t_{ij})_{i=1}^{n_i}$.
- \bowtie_{ij} represents **predicates** on terms defined by provability of goals $s \bowtie_{ij} t$ with respect to a *first-order theories* $\text{Th}_{\bowtie_{ij}}$.
- \bowtie_{ij} can be one of the following predicates:
 - One (CS-)rewriting step (\rightarrow , $\backslash\rightarrow$).
 - Zero or more (CS-)rewriting steps (\rightarrow^* , $\backslash\rightarrow^*$).
 - One or more (CS-)rewriting steps (\rightarrow^+ , $\backslash\rightarrow^+$).
 - Subterm ($|>=$) and strict subterm ($|>$).
 - (CS-)Joinability ($\rightarrow^* \leftarrow$, $\backslash\rightarrow^* \leftarrow /$).
 - One (CS-)convertibility step (\leftrightarrow , \leftarrow / \rightarrow).
 - Zero or more (CS-)convertibility steps (\leftrightarrow^* , $\leftarrow / \rightarrow^*$).

Implementation

- The tool is available here:
<http://zenon.dsic.upv.es/infChecker/>.
- It is written in Haskell and provides a first implementation of the **Feasibility Framework**, where four **processors** have been implemented:
 - P^{Sat} integrates a satisfiability approach to **prove infeasibility using model generators** as AGES and Mace4 to find a proof.
 - P^{UR} **simplifies** problems by removing non-usable rules.
 - P^{Prov} integrates a logic-based approach to program analysis to **prove feasibility by theorem proving**. In infChecker, we use the theorem prover Prover9.
 - P^{NC} adapt the processor that **narrow conditions** in the 2D DP framework for proving operational termination of CTRs to be used with feasibility sequences.

- Our **proof strategy** is:
 - ① we apply P^{UR} whenever it is sound and complete;
 - ② we try to prove feasibility using P^{Prov} ;
 - ③ if P^{Prov} fails, we apply P^{Sat} ;
 - ④ if P^{Sat} fails, we apply P^{NC} ;
 - ⑤ if P^{NC} succeeds and modifies the feasibility sequence, we repeat the strategy, otherwise we return `MAYBE`.
- Bibliography:
 - GL20** R. Gutiérrez and S. Lucas. Automatically Proving and Disproving Feasibility Conditions. In Proc. of IJCAR'2020, LNCS 12167:416–435. Springer, 2020.
 - Luc19** S. Lucas. Proving semantic properties as first-order satisfiability. Artificial Intelligence 277, paper 103174, 24 pages, 2019.
 - LG18** S. Lucas and R. Gutiérrez. Use of Logical Models for Proving Infeasibility in Term Rewriting. Information Processing Letters, 136:90-95, 2018.

2022 Results



2022 Results



- previous winners: CO3 (2014) ConCon (2015 2016 2017 2018 2020) ACP (2019)
CONFident (2021)

2022 Results



- previous winners: CO3 (2014) ConCon (2015 2016 2017 2018 2020) ACP (2019)
CONFident (2021)
- 2023 participants: ACP CO3 CONFident

CO3 (Ver. 2.4)

a COnverter for proving COnfluence of COnditional TRSs

Naoki Nishida Misaki Kojima Ayuka Matsumi
Nagoya University, Japan

Overview

CO3 proves confluence of 3-DCTRSs or infeasibility of conditions by using

- very simple termination/confluence criteria for TRSs,
- the improved sequential **unraveling** \mathcal{U}_{conf} [Gmeiner et al, 13],
- **narrowing trees** [Nishida & Maeda, 18], and
- reduction of confluence of join or semi-equational CTRSs to that of oriented ones

Infeasibility and Confluence Criterion

- Condition c is **infeasible** w.r.t. DCTRS \mathcal{R} if $\mathcal{U}_{conf}(\mathcal{R})$ is right-linear and a **narrowing tree for c** defines \emptyset [Maeda et al, 19]
- Syntactically deterministic 3-CTRSs \mathcal{R} is **confluent** if either
 - ▶ \mathcal{R} is weakly left-linear and $\mathcal{U}_{conf}(\mathcal{R})$ is confluent [Gmeiner et al, 13]
 - or
 - ▶ $\mathcal{U}_{conf}(\mathcal{R})$ is terminating and right-linear
and $\forall \langle s, t \rangle \leftarrow c \in CP(\mathcal{R}), (c = \epsilon \wedge s = t) \vee$ “ c is **infeasible**” [Maeda et al, 19]

- Improved removal of valid conditions, and strengthened disproof of CR [Ver. 2.4]

2022 Results



2022 Results



- previous winner: AGCP (2015 2016 2017 2018 2019 2020 2021)

2022 Results



- previous winner: AGCP (2015 2016 2017 2018 2019 2020 2021)
- 2023 participants: AGCP FORT-h FORT-h + FORTify

AGCP (Automated Ground Confluence Prover)

Takahito Aoto

A ground confluence prover for many-sorted TRSs

- An entrant of **GCR** category
- Written in Standard ML of New Jersey (SML/NJ)
- Methods:
 - rewriting induction for bounded convertibility
 - equivalent transformations, disproving methods
- Based on:

Improvements of the Rewriting Induction Approach for proving GCR, Aoto/Toyama/Kimura, FSCD 2017

We couldn't make any efforts on AGCP this year.

2022 Results



- ACP had most NO answers

2022 Results



- ACP had most NO answers
- previous winners: FORT (2019) ACP (2020) CoLL (2021)

2022 Results



- ACP had most NO answers
- previous winners: FORT (2019) ACP (2020) CoLL (2021)
- 2023 participants: ACP **ACP+CeTA** CoLL FORT-h FORT-h+FORTify

CoLL: Commutation Tool for Left-Linear TRSs

$\{(\mathcal{R}, \mathcal{S})\}$

CoLL

author: Kiraku Shintani (JAIST)

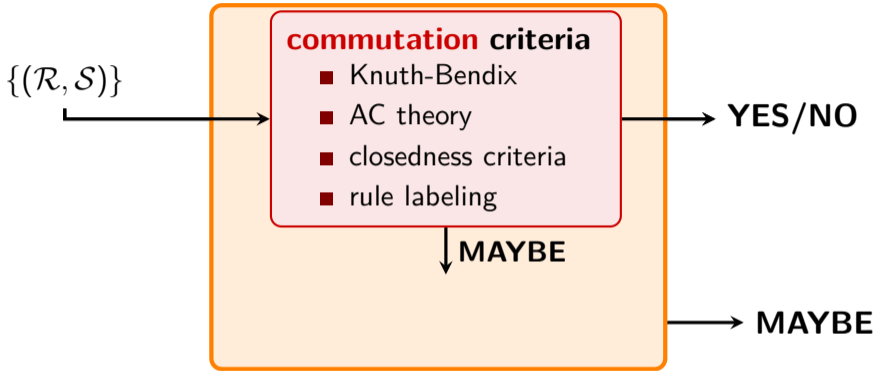
version: 1.6.1

code: OCaml (6000 LoC)

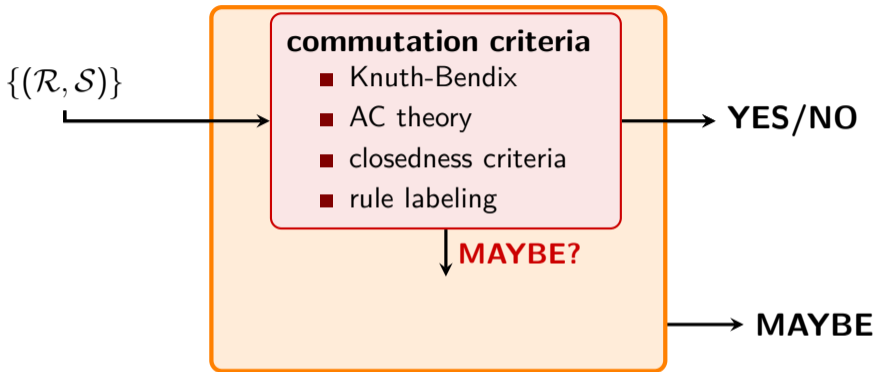
→ YES/NO

→ MAYBE

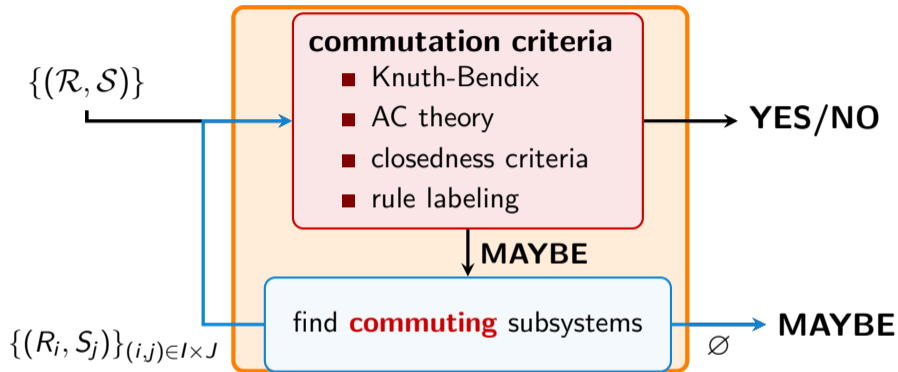
CoLL: **C**ommutation Tool for **L**eft-**L**inear TRSs



CoLL: **C**ommutation Tool for **L**eft-**L**inear TRSs



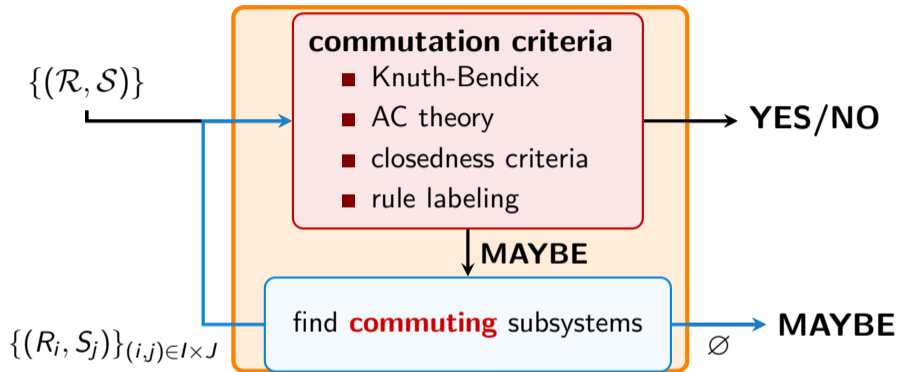
CoLL: **C**ommutation **T**ool for **L**eft-**L**inear TRSs



Theorem (Hindley 1964)

$\bigcup_{i \in I} \mathcal{R}_i$ and $\bigcup_{j \in J} \mathcal{S}_j$ commute if \mathcal{R}_i and \mathcal{S}_j commute for all $i \in I$ and $j \in J$

CoLL: **C**ommutation **T**ool for **L**eft-**L**inear TRSs

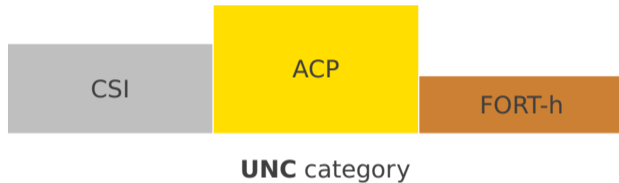


Theorem (Hindley 1964)

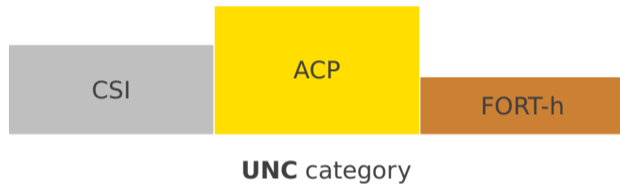
$\bigcup_{i \in I} \mathcal{R}_i$ and $\bigcup_{j \in J} \mathcal{S}_j$ commute if \mathcal{R}_i and \mathcal{S}_j commute for all $i \in I$ and $j \in J$

► **left-linearity** is often essential for commutation! ◀

2022 Results

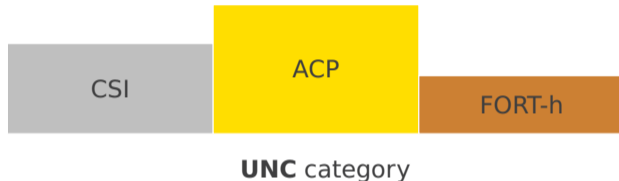


2022 Results



- previous winners: CSI (2017) ACP (2018 2019 2020 2021)

2022 Results



- previous winners: CSI (2017) ACP (2018 2019 2020 2021)
- 2023 participants: ACP CSI FORT-h FORT-h + FORTify



FORT-h 2.0

Fabian Mitterwallner

Aart Middeldorp

University of Innsbruck

FORT-h



property



linear variable-separated



TRS

$$\begin{aligned} \forall s \exists t (s \rightarrow^* t \wedge \neg \exists u (t \rightarrow u)) \\ \implies \exists v (s \dashv\vdash v \vee v \xrightarrow{\epsilon} t) \end{aligned}$$



yes | no | ?

property is arbitrary formula in first-order theory of rewriting

CoCo 2022 Categories

GCR

NFP

UNC

UNR

COM

CoCo 2022 Categories

GCR

NFP

UNC

UNR

COM

most YES results

CoCo 2023 Categories

GCR

NFP

UNC

UNR

COM

TRS

CoCo 2023 Categories

GCR NFP UNC UNR COM TRS

Literature

Aart Middeldorp, Alexander Lochmann and Fabian Mitterwallner

First-Order Theory of Rewriting for Linear Variable-Separated Rewrite Systems: Automation, Formalization, Certification

Journal of Automated Reasoning 67(2), 2023

doi: [10.1007/s10817-023-09661-7](https://doi.org/10.1007/s10817-023-09661-7)

[https://fortissimo.uibk.ac.at/fort\(ify\)/](https://fortissimo.uibk.ac.at/fort(ify)/)

2022 Results



- FORT-h and FORT-h + FORTIFY had most YES answers

2022 Results



- FORT-h and FORT-h + FORTIFY had most YES answers
- previous winner: CSI (2017 2018 2019 2020 2021)

2022 Results



- FORT-h and FORT-h + FORTIFY had most YES answers
- previous winner: CSI (2017 2018 2019 2020 2021)
- 2023 participants: **ACP** CSI FORT-h FORT-h + FORTify

ACP (Automated Confluence Prover)

Takahito Aoto

- ACP entered to **COM/CTRS/SRS/TRS/UNR/UNC**
- ACP+CeTA entered to **COM/SRS/TRS**
- Written in Standard ML of New Jersey (SML/NJ)
- Version: 0.10 (2009) ... 0.71 (2023)
- Implementing **multiple direct methods** and **divide-and-conquer methods**
- New features:
 - (i) **UNR**: preliminary efforts for proving UNR
 - (ii) **COM**: generating some certificates
- Future plan:
 - (i) **CTRS**: we're working on the implementation of:

A Critical Pair Criterion for Level-Commutation of Conditional Term Rewriting Systems, R. Haga et al., FroCoS 2023, to appear.

2022 Results



2022 Results



- previous winner: CSI (2017 2018 2019 2020 2021)

2022 Results



- previous winner: CSI (2017 2018 2019 2020 2021)
- 2023 participants: CSI FORT-h FORT-h+ FORTify

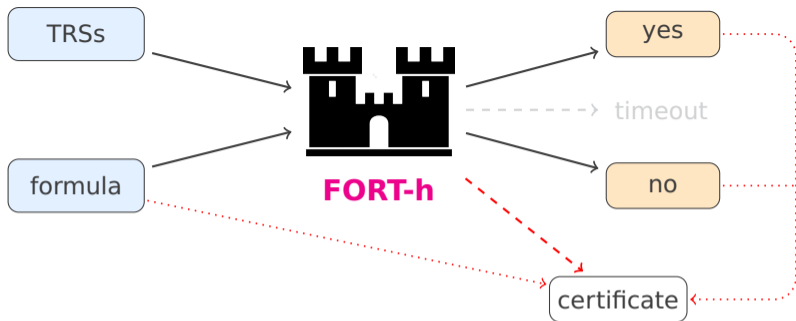


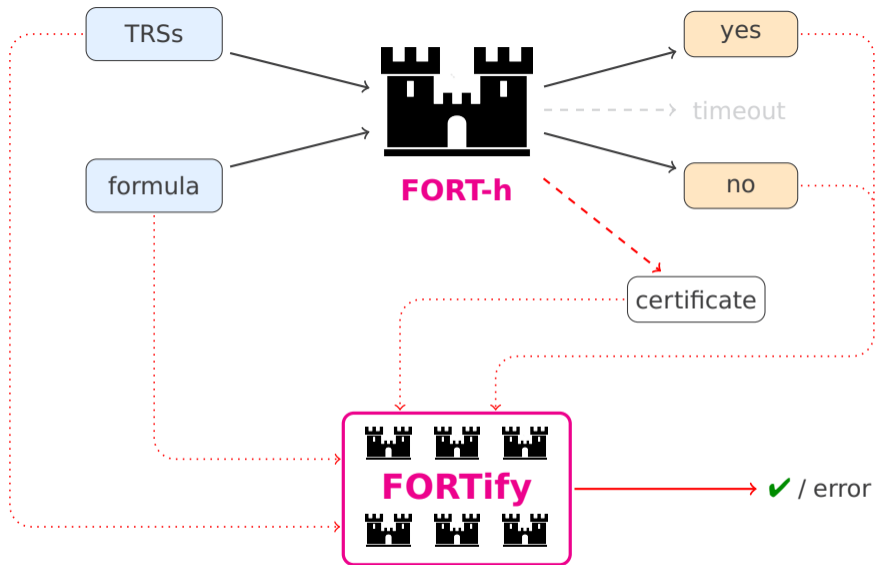
FORTify 2.0

Fabian Mitterwallner

Aart Middeldorp

University of Innsbruck





- ▶ code generated from formalization of decision procedure in Isabelle/HOL
- ▶ `https://fortissimo.uibk.ac.at/fort\(ify\)/`

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- ▶ [https://fortissimo.uibk.ac.at/fort\(ify\)/](https://fortissimo.uibk.ac.at/fort(ify)/)

CoCo 2023 Categories

COM

GCR

NFP

TRS

UNC

UNR

(with FORT-h)

- ▶ code generated from formalization of decision procedure in Isabelle/HOL
- ▶ [https://fortissimo.uibk.ac.at/fort\(ify\)/](https://fortissimo.uibk.ac.at/fort(ify)/)

CoCo 2023 Categories

COM GCR NFP TRS UNC UNR (with FORT-h)

Literature

Aart Middeldorp, Alexander Lochmann and Fabian Mitterwallner

First-Order Theory of Rewriting for Linear Variable-Separated Rewrite Systems: Automation, Formalization, Certification

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doi: [10.1007/s10817-023-09661-7](https://doi.org/10.1007/s10817-023-09661-7)

2022 Results



2022 Results



- 2023 participants: **ConfCSR** CONFident



CoCo 2023 Participant: ConfCSR

Filip Stevanovic and Fabian Mitterwallner

ConfCSR

- tool for automatically checking (non-)confluence of context-sensitive rewrite systems
- participates in the CSR category
- written in Haskell as a bachelor project
- open-source:

`https://github.com/F200907/ConfCSR`

- relies on external termination checker such as AProVE2
- does currently not support conditional rewriting

and the 2023 winners are ...



<http://cocograph.uibk.ac.at/2023.html>

Outline

1. Acknowledgements

2. History

3. 2023

4. Awards

5. Outlook

The logo consists of three brown, textured circles arranged horizontally, resembling cocoa beans.

the steering committee of the

12th Confluence Competition

awarded

TOOL by *TOOL* authors

the

First Place in the ? category

Obergurgl, 23 August 2023

Awards

- top three tools in each category (excluding tool + certifier)

Awards

- top three tools in each category (excluding tool + certifier)
- certifiers based on total number of certified answers

CERTIFICATION

Awards

- top three tools in each category (excluding tool + certifier)
- certifiers based on total number of certified answers
- top three tools that produce most certifiable answers

CERTIFICATION

RELIABILITY

Awards

- top three tools in each category (excluding tool+certifier)
- certifiers based on total number of certified answers
- top three tools that produce most certifiable answers

CERTIFICATION

RELIABILITY

Award Ceremony during IWC business meeting (tomorrow 12:00 – 12:30)

Outline

1. Acknowledgements
2. History
3. 2023
4. Awards
- 5. Outlook**

Outlook

- CoCo 2024 during IWC 2024 ?

Outlook

- CoCo 2024 during IWC 2024 ?
- new T-shirt ?

Outlook

- CoCo 2024 during IWC 2024 ?
- new T-shirt ?
- new category on confluence of logically constrained rewrite systems

LCTRS

Outlook

- CoCo 2024 during IWC 2024 ?
- new T-shirt ?
- new category on confluence of logically constrained rewrite systems
- new format and infrastructure

LCTRS



Outlook

- CoCo 2024 during IWC 2024 ?
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LCTRS

