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where $\mathcal{R}\restriction_{\mathcal{C}} = \{\ell \to r \in \mathcal{R} \mid \mathcal{F}\mathsf{un}(\ell) \subseteq \mathcal{F}\mathsf{un}(\mathcal{C})\}$

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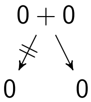

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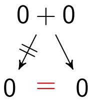

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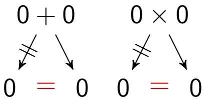

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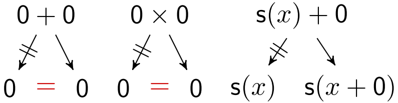

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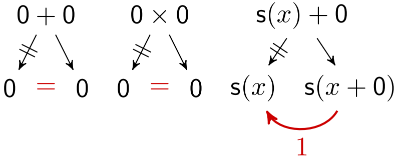

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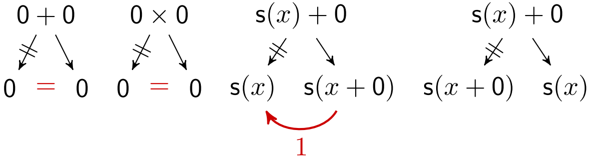

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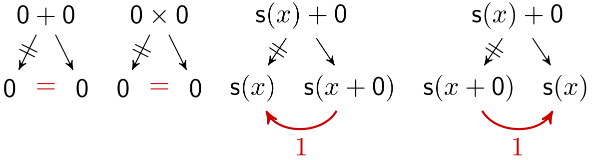

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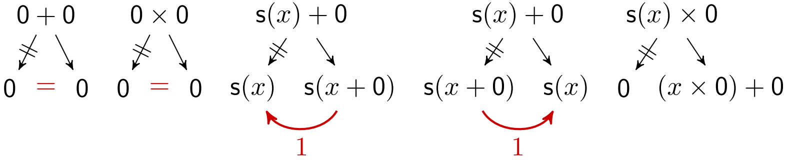

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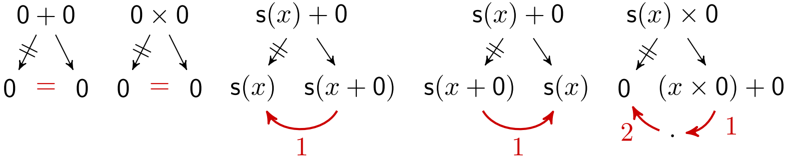

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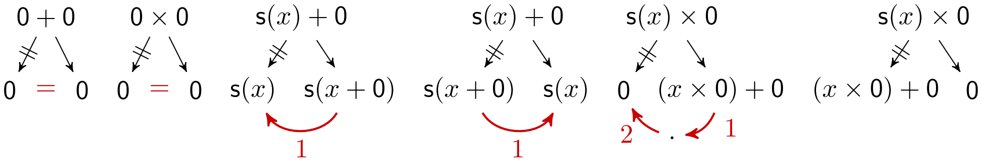

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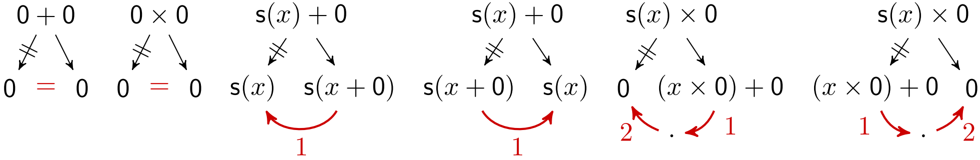

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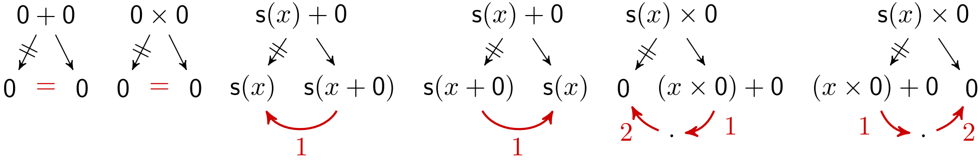

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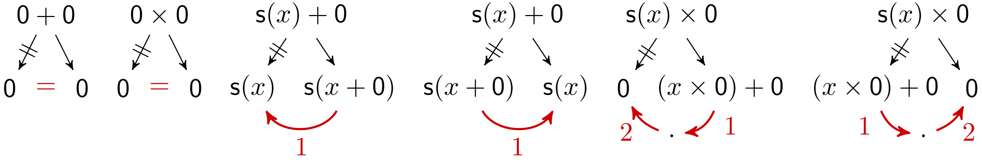

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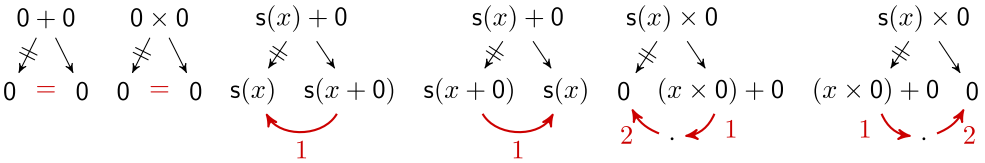

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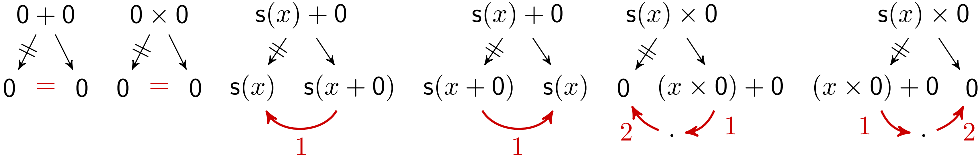

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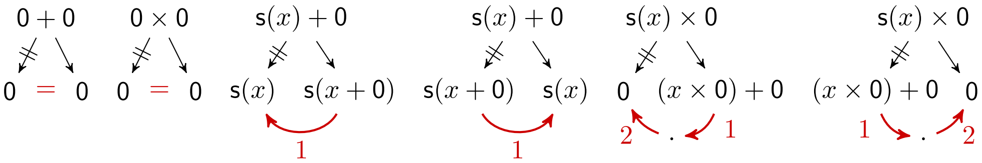

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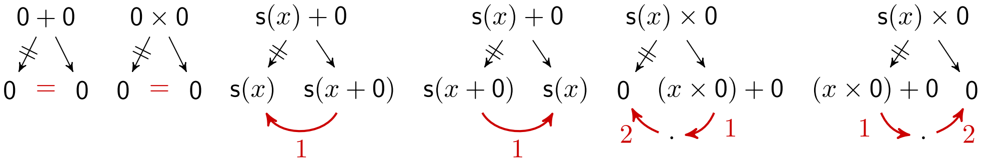

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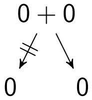

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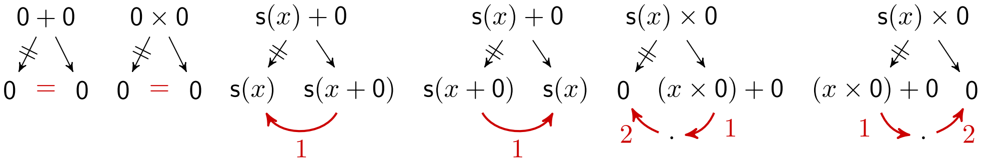

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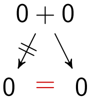

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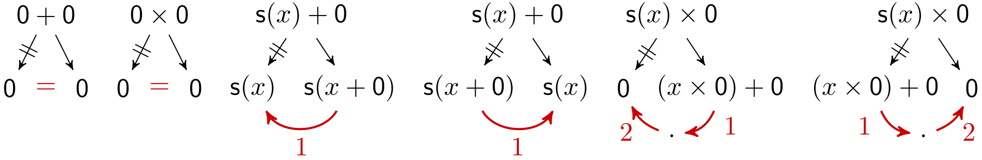

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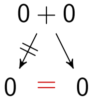

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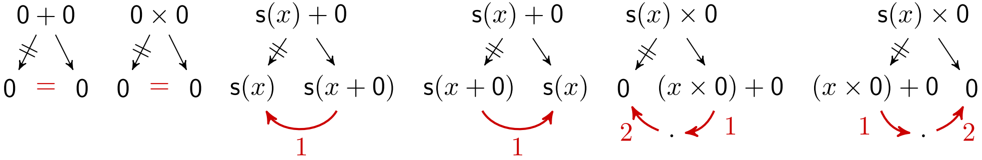

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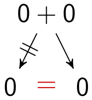

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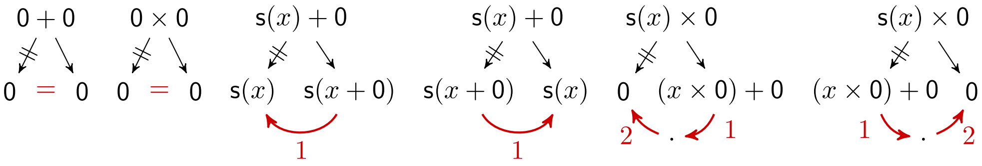

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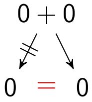

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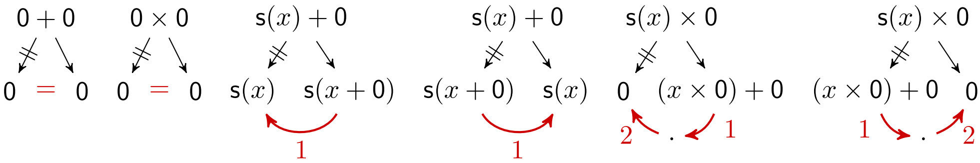

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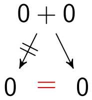


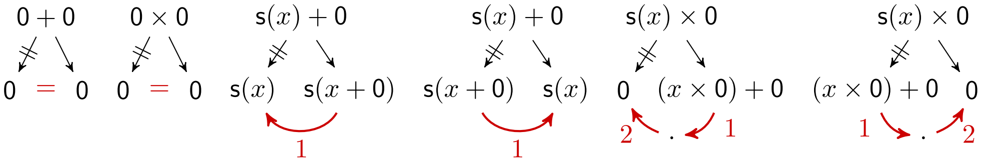
and subsystem over $\mathcal{F}\mathsf{un}(\varnothing) = \varnothing$ is $\varnothing$

[3] $\mathsf{CR}(\varnothing)$ is trivial,

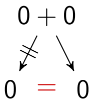
using reduction method, we show confluence of left-linear TRS $\mathcal{R}$:

$$\begin{array}{llllll} 1: & x + 0 \to x & 3: & 0 + y \to y & 5: & \mathsf{s}(x) + y \to \mathsf{s}(x + y) \\ 2: & x \times 0 \to 0 & 4: & \mathsf{s}(x) \times 0 \to 0 & 6: & \mathsf{s}(x) \times y \to (x \times y) + y \end{array}$$

[1] $\mathsf{CR}(\mathcal{R}) \iff \mathsf{CR}(\{1,2,3\})$ because



and subsystem over $\mathcal{F}\mathsf{un}(\{1,2\}) = \{0, +, \times\}$ is $\{1,2,3\}$

[2] $\mathsf{CR}(\{1,2,3\}) \iff \mathsf{CR}(\varnothing)$ because  and subsystem over $\mathcal{F}\mathsf{un}(\varnothing) = \varnothing$ is $\varnothing$

[3] $\mathsf{CR}(\varnothing)$ is trivial, hence $\mathsf{CR}(\mathcal{R})$