Hakusan 0.8: A Confluence Tool

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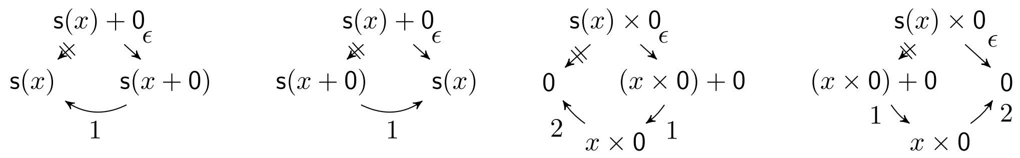
Hakusan (`http://www.jaist.ac.jp/project/saigawa/`) is a confluence tool for left-linear term rewrite systems (TRSs). It analyzes confluence by using the two *compositional* confluence criteria [2, Theorems 31 and 38] that originate from rule labeling and critical pair systems. This version supports two new features. One is certificate outputs for rule labeling [2, Theorem 28] which are verifiable by CeTA [3], and the other is the following *reduction method* for confluence problems (see the extended version of [2]). Let $\mathcal{R}\restriction_{\mathcal{C}} = \{\ell \to r \in \mathcal{R} \mid \mathcal{F}\mathsf{un}(\ell) \subseteq \mathcal{F}\mathsf{un}(\mathcal{C})\}$.

Theorem 1. *Let $\mathcal{C}$ be a subsystem of a left-linear TRS $\mathcal{R}$. Suppose ${}_{\mathcal{R}}{+\!\!\!+\!\!\!\!\leftarrow} \rtimes \xrightarrow{\epsilon}_{\mathcal{R}} \subseteq \leftrightarrow^*_{\mathcal{C}}$ and $\mathcal{R}\restriction_{\mathcal{C}} \subseteq \to^*_{\mathcal{C}}$. The TRS $\mathcal{R}$ is confluent if and only if $\mathcal{C}$ is confluent.*

To demonstrate the reduction method, we show the confluence of the left-linear TRS $\mathcal{R}$:

$$
\begin{array}{lll}
1\colon\ x + 0 \to x & 3\colon\ \ 0 + y \to y & 5\colon\ \mathsf{s}(x) + y \to \mathsf{s}(x + y) \\
2\colon\ x \times 0 \to 0 & 4\colon\ \mathsf{s}(x) \times 0 \to 0 & 6\colon\ \mathsf{s}(x) \times y \to (x \times y) + y
\end{array}
$$

There are four non-trivial parallel critical pairs and they admit the following diagrams:



(i) Let $\mathcal{C} = \{1, 2, 3\}$. We have ${}_{\mathcal{R}}{+\!\!\!+\!\!\!\!\leftarrow} \rtimes \xrightarrow{\epsilon}_{\mathcal{R}} \subseteq \leftrightarrow^*_{\mathcal{C}}$. As $\mathcal{F}\mathsf{un}(\mathcal{C}) = \{0, +, \times\}$, the inclusion $\mathcal{R}\restriction_{\mathcal{C}} = \{1, 2, 3\} \subseteq \to^*_{\mathcal{C}}$ holds. According to Theorem 1, the confluence problem of $\mathcal{R}$ is reduced to that of $\mathcal{C}$.

(ii) Since $\mathcal{C}$ only admits a trivial parallel critical pair, it is closed by the empty system $\varnothing$. Moreover, the inclusion $\mathcal{C}\restriction_{\varnothing} = \varnothing \subseteq \to^*_{\varnothing}$ holds. Hence, by Theorem 1 confluence of $\mathcal{C}$ is reduced to that of the empty system $\varnothing$.

(iii) Since the empty system $\varnothing$ is trivially confluent, we conclude that $\mathcal{R}$ is confluent.

As a final remark, our tool employs the SMT solver Z3 [1] and the termination tool NaTT [4] for automating the compositional confluence criteria and the reduction method.

References

[1] L. de Moura and N. Bjørner. Z3: An Efficient SMT Solver. In *Proc. 12th TACAS*, volume 4963 of LNCS, pages 337–340, 2008.

[2] K. Shintani and N. Hirokawa. Compositional Confluence Criteria. In *Proc. 7th FSCD*, volume 228 of *LIPIcs*, pages 28:1–28:19, 2022. The extended version, submitted to a journal, is available at: doi: 10.48550/arXiv.2303.03906

[3] R. Thiemann and C. Sternagel. Certification of Termination Proofs using CeTA. In *Proc. 22nd TPHOLs*, volume 5674 of LNCS, pages 452–468, 2009.

[4] A. Yamada and K. Kusakari and T. Sakabe. Nagoya Termination Tool. In *Proc. 25th RTA*, volume 8560 of *LNCS*, pages 446–475, 2014.