

# CO3 (Version 2.4)

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CO3, a converter for proving confluence of conditional TRSs,<sup>1</sup> tries to prove confluence of conditional term rewrite systems (CTRSs, for short) by using a transformational approach (cf. [7]). The tool first transforms a given weakly-left-linear (WLL, for short) 3-DCTRS into an unconditional term rewrite system (TRS, for short) by using  $\mathbb{U}_{conf}$  [3], a variant of the *unraveling*  $\mathbb{U}$  [9], and then verifies confluence of the transformed TRS by using the following theorem: A 3-DCTRS  $\mathcal{R}$  is confluent if  $\mathcal{R}$  is WLL and  $\mathbb{U}_{conf}(\mathcal{R})$  is confluent [2, 3]. The tool is very efficient because of very simple and lightweight functions to verify properties such as confluence and termination of TRSs.

Since version 2.0, a *narrowing-tree*-based approach [8, 4] to prove infeasibility of a condition w.r.t. a CTRS has been implemented [5]. The approach is applicable to *syntactically deterministic* CTRSs that are operationally terminating and *ultra-right-linear* w.r.t. the *optimized* unraveling. To prove infeasibility of a condition  $c$ , the tool first prove confluence, and then linearizes  $c$  if failed to prove confluence; then, the tool computes and simplifies a narrowing tree for  $c$ , and examines the emptiness of the narrowing tree. Since version 2.2, CO3 accepts both *join* and *semi-equational* CTRSs, and transforms them into equivalent DCTRSs to prove confluence or infeasibility [6].

This version has an improvement on the removal of valid conditions: For a conditional rule  $\ell \rightarrow r \Leftarrow c, s \rightarrow t, c' \in \mathcal{R}$ , if there exist an unconditional rule  $\ell' \rightarrow r' \in \mathcal{R}$  and a substitution  $\theta$  such that  $\ell'\theta = s$  and  $r'\theta = t$ , the condition  $s \rightarrow t$  is dropped from the conditional part, replacing the rule by  $\ell \rightarrow r \Leftarrow c, c'$ . In addition, we slightly strengthen the function to disprove confluence: In proving strong irreducibility of a term  $t$ , if a subterm  $u$  of  $t$  is unifiable with the left-hand side of a rule  $\ell \rightarrow r \Leftarrow c$  by means of an mgu  $\theta$ , then we check infeasibility of  $c\theta$ ; if  $c\theta$  is infeasible, then the rule is considered to be inapplicable to  $u$ .

## References

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<sup>1</sup><http://www.trs.css.i.nagoya-u.ac.jp/co3/>