Hakusan 0.5: A Confluence Tool

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Hakusan is a prototype tool for automatically proving confluence of left-linear term rewrite systems (TRSs). The tool, written in Haskell, is freely available at:

http://www.jaist.ac.jp/project/saigawa/

The typical usage is: `hakusan <file>`. Here the input file is written in the TRS format [3]. The tool outputs `YES` if confluence of the input TRS is proved, and `MAYBE` if the tool does not reach any conclusion. Currently the tool does not support non-confluence analysis.

Confluence analysis in Hakusan is based on *compositional* confluence criteria [4], which mean sufficient conditions such that, given a rewrite system $\mathcal{R}$ and its subsystem $\mathcal{C} \subseteq \mathcal{R}$, confluence of $\mathcal{C}$ implies that of $\mathcal{R}$. Compositional criteria can be seen as a combination method for confluence analysis. Hakusan alternately uses two compositional confluence criteria: One is a compositional version of the rule labeling method [6, Theorem 56], and the other is a compositional version of the confluence criterion by critical pair systems [1].

Theorem 1. *Let $\mathcal{R}$ be a left-linear TRS and $\mathcal{C}$ a confluent TRS with $\mathcal{C} \subseteq \mathcal{R}$, and also let ϕ and ψ be labeling functions from $\mathcal{R}$ to $\mathbb{N}$. The TRS $\mathcal{R}$ is confluent if we have $\mathcal{R}_{\phi,0} = \mathcal{C} = \mathcal{R}_{\psi,0}$ and the following conditions hold for all $(k, m) \in \mathbb{N}^2 \setminus \{(0, 0)\}$.*

- *Every parallel critical peak of form $t \;{}_{\phi,k}\!\twoheadleftarrow\; s \xrightarrow{\epsilon}_{\psi,m} u$ is (ψ, ϕ)-decreasing.*
- *Every parallel critical peak of form $t \;{}_{\psi,m}\!\twoheadleftarrow\; s \xrightarrow{\epsilon}_{\phi,k} u$ is (ϕ, ψ)-decreasing.*

Here $\mathcal{R}_{\phi,k}$ stands for $\{\ell \to r \in \mathcal{R} \mid \phi(\ell \to r) \leqslant k\}$ and $\twoheadrightarrow_{\phi,k}$ for the parallel step of $\mathcal{R}_{\phi,k}$. See [4, Definition 27] for the definition of (ψ, ϕ)-decreasingness.

Theorem 2. *Let $\mathcal{R}$ be a left-linear TRS and $\mathcal{C}$ a confluent TRS with $\mathcal{C} \subseteq \mathcal{R}$. The TRS $\mathcal{R}$ is confluent if ${}_{\mathcal{R}}\!\twoheadleftarrow \rtimes \xrightarrow{\epsilon}_{\mathcal{R}} \subseteq \to_{\mathcal{R}}^* \cdot {}_{\mathcal{R}}^*\!\leftarrow$ and $\mathcal{P}/\mathcal{R}$ is terminating. Here $\mathcal{P}$ stands for the TRS: $\{s \to t, s \to u \mid t \;{}_{\mathcal{R}}\!\twoheadleftarrow\; s \xrightarrow{\epsilon}_{\mathcal{R}} u$ is a parallel critical peak but not $t \leftrightarrow_{\mathcal{C}}^* u\}$.*

For automation, the tool employs the SMT solver Z3 [2] for finding suitable labeling functions, and the termination tool NaTT [5] for testing relative termination.

References

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