CoCo 2021 Participant: CeTA 2.40[*]

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The tool CeTA [1] is a certifier for, among other properties, (non-)confluence of term rewrite systems with and without conditions. Its soundness is proven as part of the formal proof library IsaFoR, the Isabelle Formalization of Rewriting. For a complete reference of supported techniques we refer to the certification problem format (CPF) and the IsaFoR/CeTA website:

<center>http://cl-informatik.uibk.ac.at/isafor/</center>

In the following, we describe the relevant changes of version 2.40 of CeTA w.r.t. confluence proving. Although there are no new dedicated confluence techniques in CeTA, we like to mention an extended support for termination proofs. This extension has the potential to increase the power of confluence techniques that rely upon termination or relative termination.

In particular there is an extension that consists of a generalization of the weighted path order (WPO) [3, 2], where we now added support for multiset comparisons (cases (2c) and (2d)). Consequently, the generalized version of WPO subsumes the recursive path order.

Definition 1 (Generalized WPO)**.** *Let $\mathcal{A}$ be a weakly monotone algebra over signature Σ, $\succsim$ a precedence, π a status, and let $c : \Sigma \to \{\mathsf{lex}, \mathsf{mul}\}$. Let $\geq_{\mathcal{A}}$ be simple w.r.t. π. The WPO reduction pair $(\succ_{\mathsf{WPO}}, \succsim_{\mathsf{WPO}})$ is defined as follows: $s \succ_{\mathsf{WPO}} t$ iff $s >_{\mathcal{A}} t$, or $s \geq_{\mathcal{A}} t$ and*

1. *$s = f(s_1, \ldots, s_n)$ and $\exists i \in \pi(f). \ s_i \succsim_{\mathsf{WPO}} t$, or*
2. *$s = f(s_1, \ldots, s_n)$, $t = g(t_1, \ldots, t_m)$, $\forall j \in \pi(g). \ s \succ_{\mathsf{WPO}} t_j$ and*

   (a) *$f \succ g$,*
   (b) *$f \succsim g$ and $c(f) = c(g) = \mathsf{lex}$ and $\pi(f)[s_1, \ldots, s_n] \succ_{\mathsf{WPO}}^{\mathsf{lex}} \pi(g)[t_1, \ldots, t_m]$,*
   (c) *$f \succsim g$ and $c(f) = c(g) = \mathsf{mul}$ and $\pi(f)[s_1, \ldots, s_n] \succ_{\mathsf{WPO}}^{\mathsf{mul}} \pi(g)[t_1, \ldots, t_m]$, or*
   (d) *$f \succsim g$ and $c(f) \neq c(g)$ and $\pi(f)[s_1, \ldots, s_n] \neq [\,]$ and $\pi(g)[t_1, \ldots, t_m] = [\,]$.*

The relation $s \succsim_{\mathsf{WPO}} t$ is defined in a similar way and we refer to theory `Orderings/WPO_MS.thy` *within* IsaFoR *for the full formal definition.*

We would like to welcome all confluence tool developers to experiment whether our new extension is indeed helpful for confluence proving, and are looking forward to certify these new kinds of proofs via CeTA.[1]

References

[1] René Thiemann and Christian Sternagel. Certification of Termination Proofs Using CeTA. In *Theorem Proving in Higher Order Logics, 22nd International Conference, Proceedings*, volume 5674 of *LNCS*, pages 452–468. 2009.

[2] René Thiemann, Jonas Schöpf, Christian Sternagel, and Akihisa Yamada. Certifying the Weighted Path Order. In *Formal Structures for Computation and Deduction, 5th International Conference, Proceedings*, volume 167 of *LIPIcs*, pages 4:1–4:20, 2020.

[3] Akihisa Yamada, Keiichirou Kusakari, and Toshiki Sakabe. A Unified Ordering for Termination Proving. *Sci. Comput. Program.*, 111:110–134, 2015.

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[1]The CPF-format has not yet been extended to cover the extension of WPO to multisets. We invite all tool authors to get in contact with us to fix the precise format for WPO with multiset comparisons.