CO3 (Version 2.2)

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CO3, a converter for proving confluence of conditional TRSs,[1] tries to prove confluence of conditional term rewrite systems (CTRSs, for short) by using a transformational approach (cf. [5]). The tool first transforms a given weakly-left-linear (WLL, for short) 3-DCTRS into an unconditional term rewrite system (TRS, for short) by using $\mathbb{U}_{conf}$ [2], a variant of the *unraveling* $\mathbb{U}$ [8], and then verifies confluence of the transformed TRS by using the following theorem: a 3-DCTRS $\mathcal{R}$ is confluent if $\mathcal{R}$ is WLL and $\mathbb{U}_{conf}(\mathcal{R})$ is confluent [1, 2]. The tool is very efficient because of very simple and lightweight functions to verify properties such as confluence and termination of TRSs. Since version 2.0, a *narrowing-tree*-based approach [6, 3] to prove infeasibility of a condition w.r.t. a specified CTRS has been implemented [4]. The approach is applicable to *syntactically deterministic* CTRSs that are operationally terminating and *ultra-right-linear* w.r.t. the *optimized* unraveling.

When *join* and *semi-equational* CTRSs are given as input, the previous version returns `MAYBE` but the present one accepts them as input. To prove confluence of join CTRSs, we consider them as oriented ones [7, Section 5.3].

Theorem 1. *Let $\mathcal{R}$ be a join CTRS, and $\mathcal{R}'$ be $\{\ell \to r \Leftarrow s_1 \twoheadrightarrow x_1, t_1 \twoheadrightarrow x_1, \ldots, s_k \twoheadrightarrow x_k, t_k \twoheadrightarrow x_k \in \mathcal{R} \mid \ell \to r \Leftarrow s_1 \downarrow t_1, \ldots, s_k \downarrow t_k,$ $x_1, \ldots, x_k$ are distinkt fresh variables$\}$. Then, (1) $\to_{\mathcal{R}} = \to_{\mathcal{R}'}$, and (2) $\mathcal{R}$ is confluent if and only if $\mathcal{R}'$ is so.*

To prove confluence of semi-equational CTRSs, we consider them as join (i.e., oriented) ones.

Theorem 2. *Let $\mathcal{R}$ be a semi-equational CTRS, and $\mathcal{R}'$ be $\{\ell \to r \Leftarrow s_1 \downarrow t_1, \ldots, s_k \downarrow t_k \in \mathcal{R} \mid \ell \to r \Leftarrow s_1 \leftrightarrow^* t_1, \ldots, s_k \leftrightarrow^* t_k\}$. Then, all of the following hold: (1) $\to_{\mathcal{R}} \supseteq \to_{\mathcal{R}'}$; if $\mathcal{R}'$ is confluent, then (2) $\to_{\mathcal{R}} \subseteq \to_{\mathcal{R}'}$ and (3) $\mathcal{R}$ is confluent.*

Note that the present version does not disprove confluence of join and semi-equational CTRSs.

To prove infeasibility of a condition c, the tool first prove confluence, and then linearizes c if failed to prove confluence; then, the tool computes and simplifies a narrowing tree for c, and examines the emptiness of the narrowing tree.

References

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