# CoLL

A Confluence Tool Based on Commutation

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# Theoretical Framework

### Definition

ARSs $\mathcal{A}$ and $\mathcal{B}$ commute if

$$\mathrel{{}_{\mathcal{A}}^{*}\!\!\leftarrow} \cdot \to_{\mathcal{B}}^{*} \;\subseteq\; \to_{\mathcal{B}}^{*} \cdot \mathrel{{}_{\mathcal{A}}^{*}\!\!\leftarrow}$$

$$\begin{tikzcd}
 & \cdot \arrow[dl, "*"'] \arrow[dr, "*"] & \\
\cdot \arrow[dr, "*"'] & & \cdot \arrow[dl, "*"] \\
 & \cdot &
\end{tikzcd}$$

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### Theorem (Commutation Lemma [Hindley 64])

*ARSs $\mathcal{A}$ and $\mathcal{B}$ commute if every $\rightarrow_\alpha$ and $\rightarrow_\beta$ commute*

*where* $\mathcal{A} = \langle A, \{\rightarrow_\alpha\}_{\alpha \in I} \rangle$ *and* $\mathcal{B} = \langle A, \{\rightarrow_\beta\}_{\beta \in J} \rangle$

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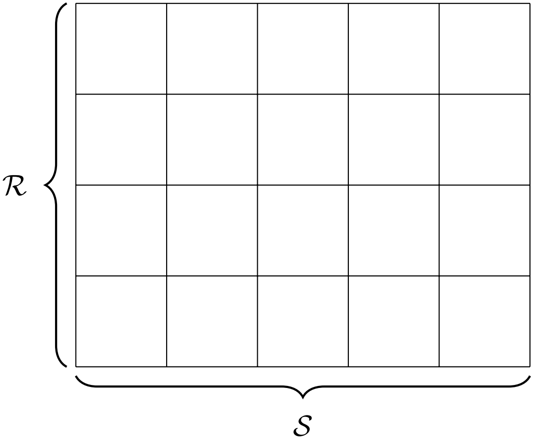
### Lemma (self commutation)

*ARS $\mathcal{A}$ is confluent if $\mathcal{A}$ and $\mathcal{A}$ commute*

# Commutation Lemma



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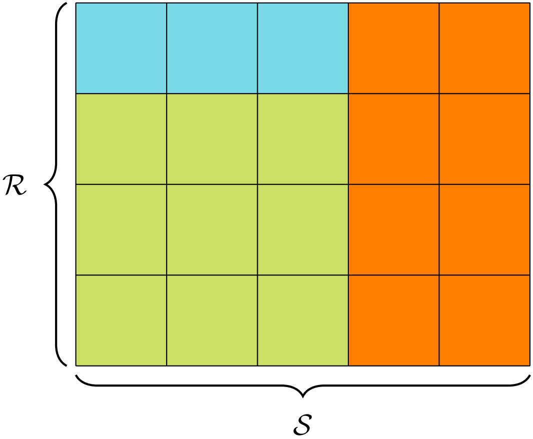

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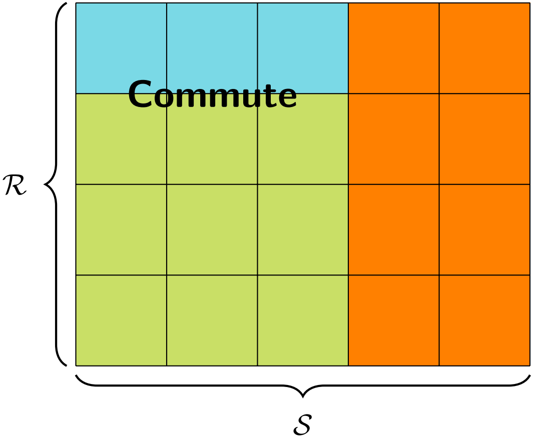

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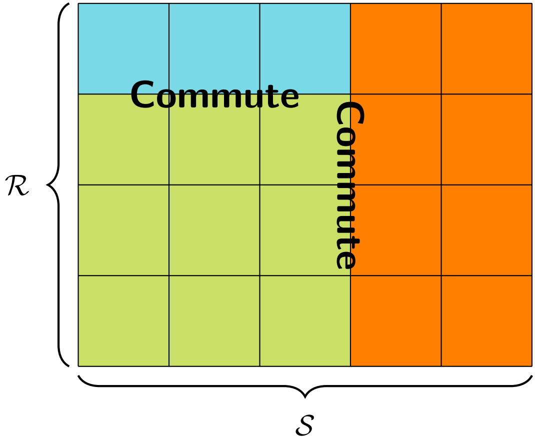

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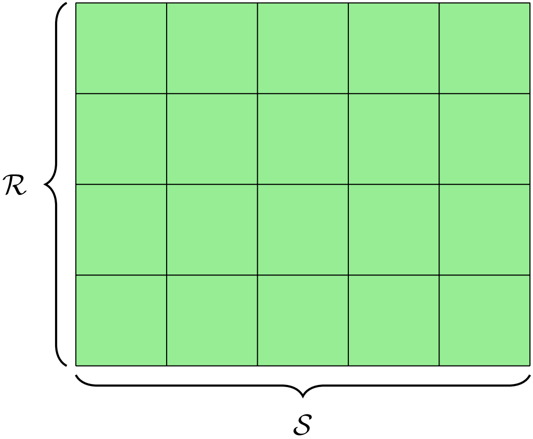

$\mathcal{R}$ and $\mathcal{S}$ commute

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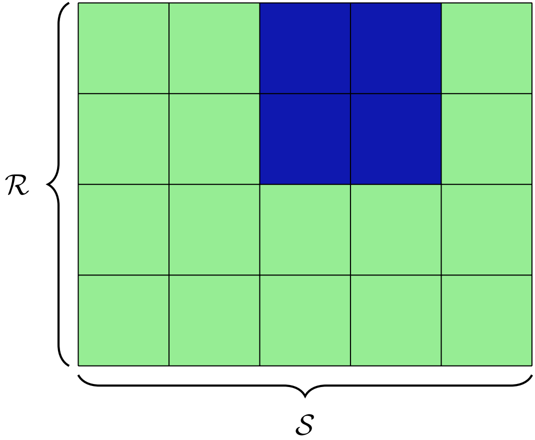

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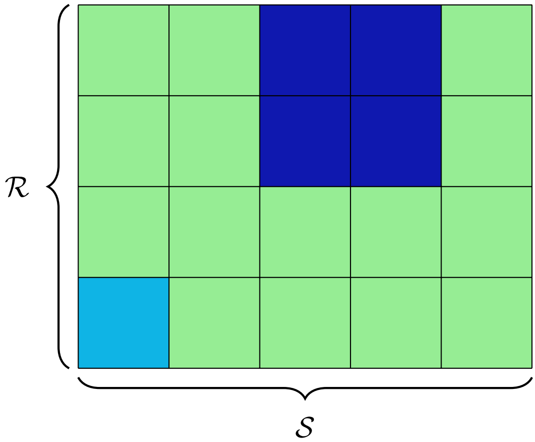

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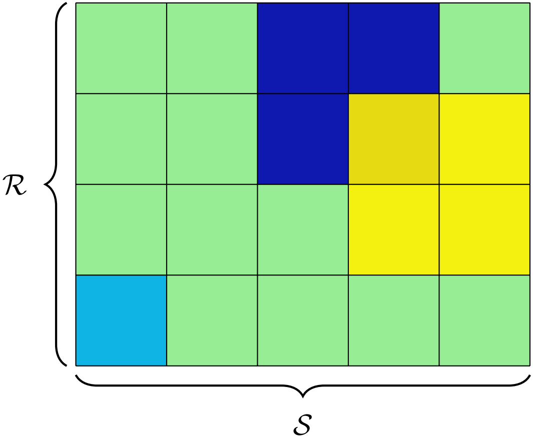

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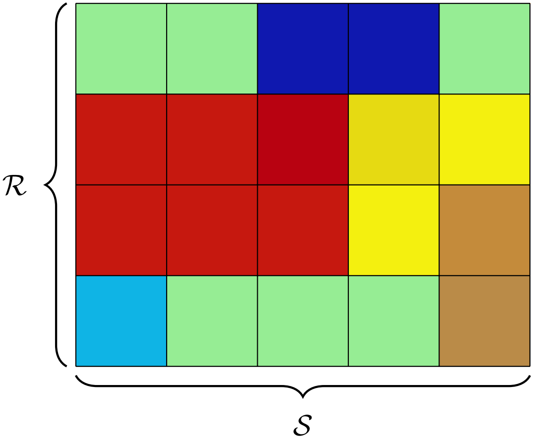

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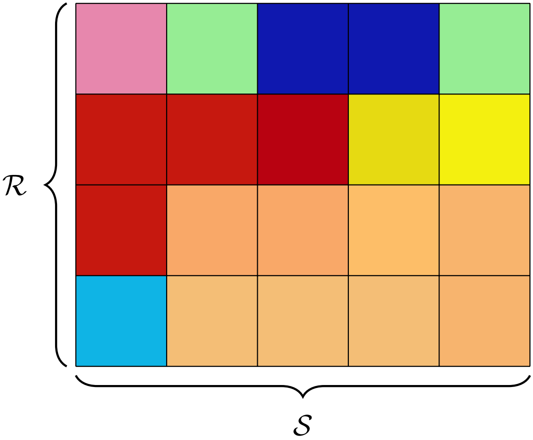

too many commutable subsystems!

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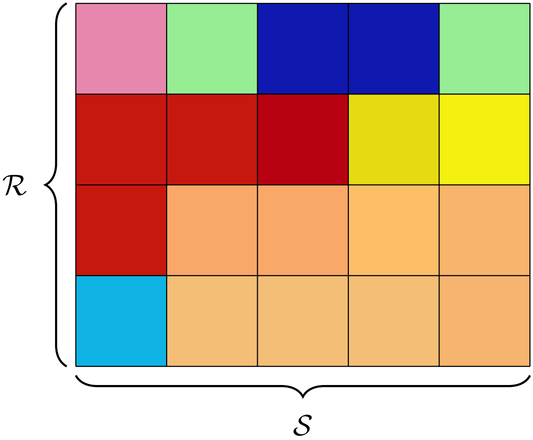

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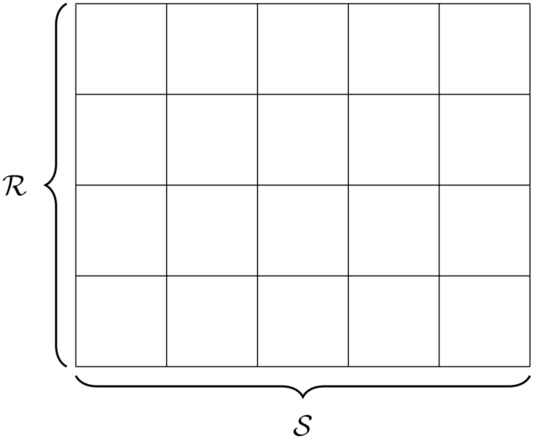
how to find suitable subsystems?

# MaxSAT

every time, take maximal commuting subsystems



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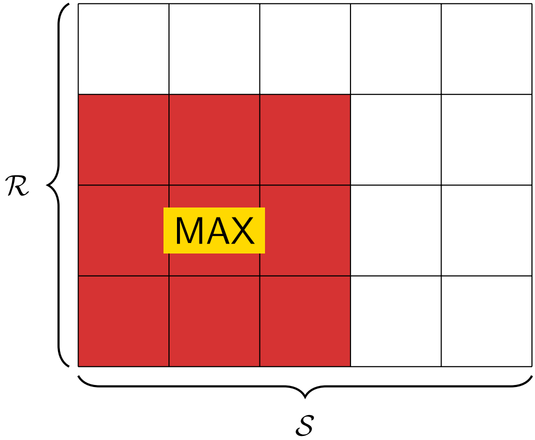

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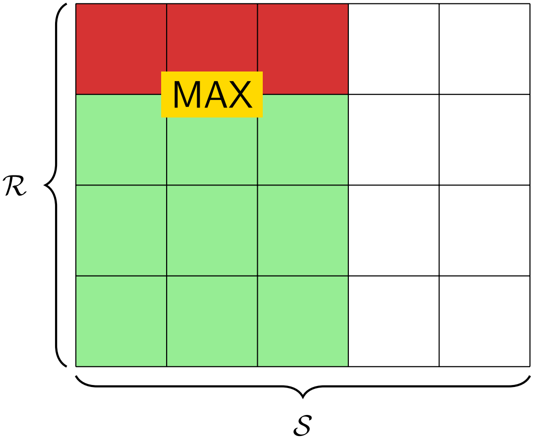

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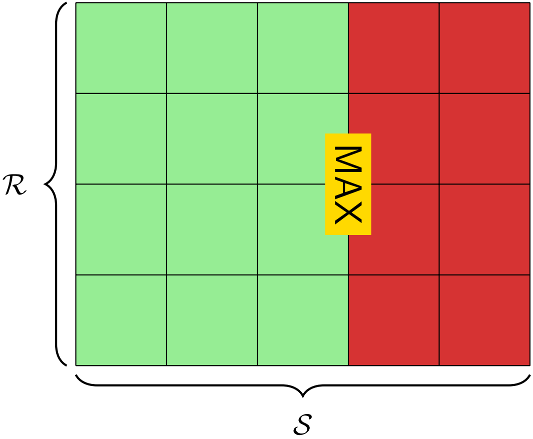

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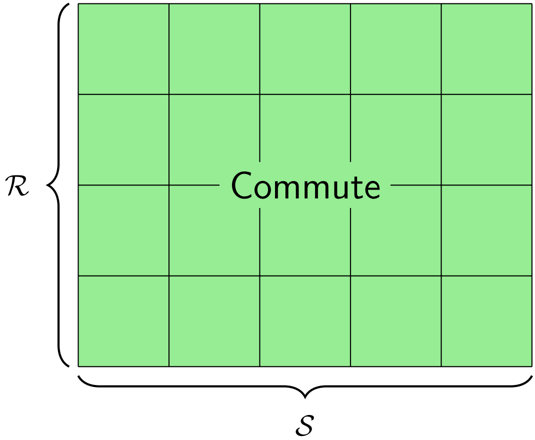

# Conclusion

picture of “Coll island” from http://en.wikipedia.org/wiki/Coll

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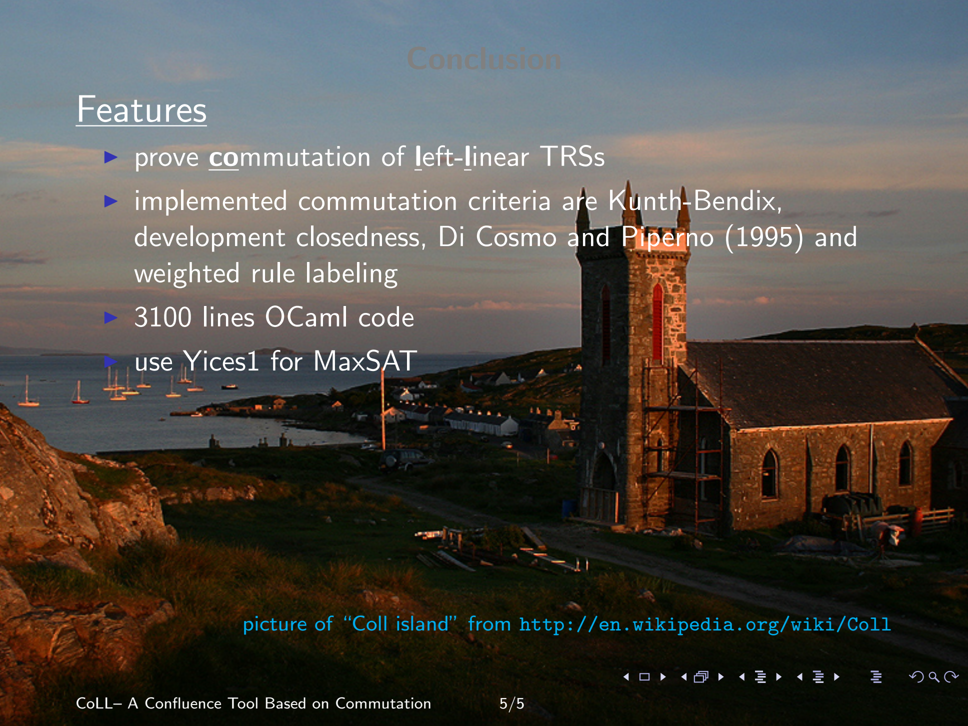
## Features

- prove **co**mmutation of **l**eft-**l**inear TRSs
- implemented commutation criteria are Kunth-Bendix, development closedness, Di Cosmo and Piperno (1995) and weighted rule labeling
- 3100 lines OCaml code
- use Yices1 for MaxSAT



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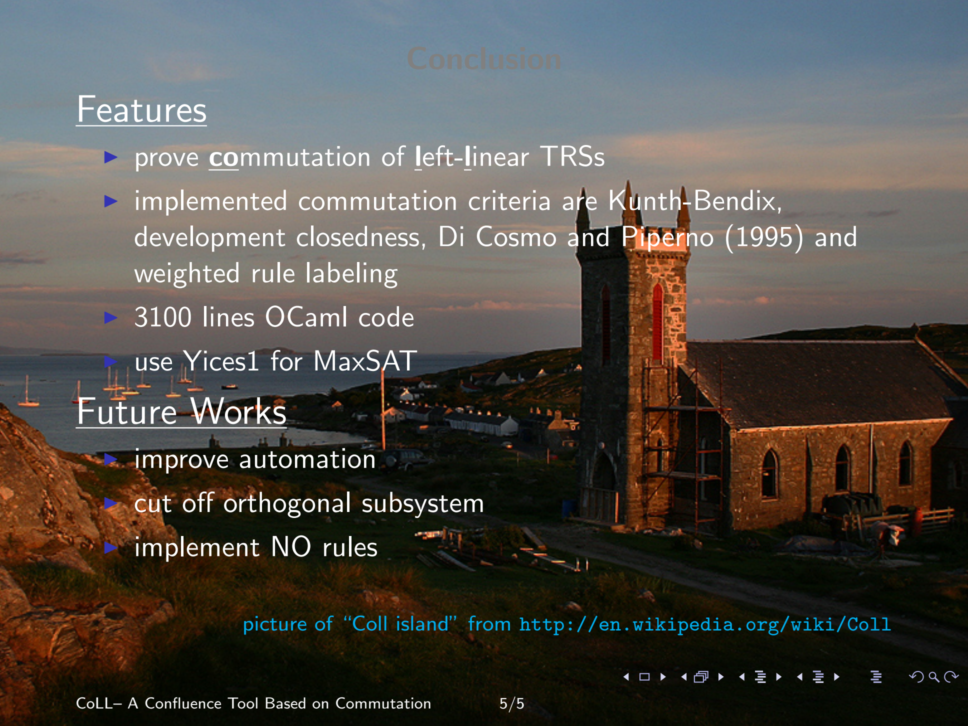

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## Future Works

- improve automation
- cut off orthogonal subsystem
- implement NO rules

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